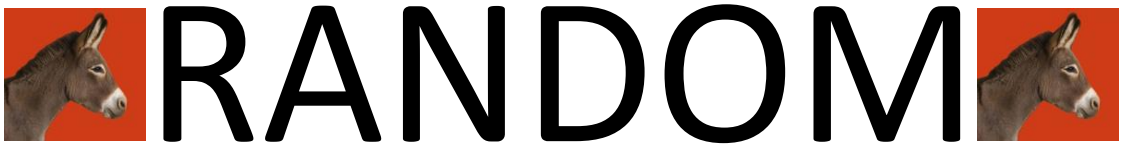
VICTORIAN CERTIFICATE OF EDUCATION

2022

STUDENT NAME:

MATHEMATICAL METHODS UNIT 4

SAC 3 (Tech-Active)



Reading Time: 15 minutes
Writing time: 120 minutes

QUESTION AND ANSWER BOOK

Number of Questions	*Number of questions to be answered*	*Number of marks*
9	9	78
		Total 78

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one laptop with Mathematica and any number of Mathematica files.
- Students are NOT permitted to bring into the examination: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 23 pages.
- Formula sheet

Instructions

- Write your name in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.

This page is blank, as always

Note: Sections between these symbols are story only and are not needed to answer the questions

In the tenth year of the Tau-Region War, ArSchmidties —a demigod and the greatest of all the Mathematical warriors—stands brooding on the shores of the sea. He thinks about his friend and adoptive brother Patrolambous.

Patrolambous has recently died, in part because of ArSchmidties's own actions. AgaKermond, the leader of the Mathematical army, had taken ArSchmidties slave from him, and ArSchmidties in turn refused to fight, angered by both the loss of the slave and the insult of AgaKermond taking them.

Eventually, however, he agreed to let Patrolambous wear his armour and lead his men in his place, resulting in Patrolambous's death at the hands of the Tau-Region prince Hectorcic.

Instructions

Answer **all** questions in the spaces provided

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (11 marks)

The time, T hours, that ArSchmidties spends brooding by the sea each day follows a normal distribution with mean $\mu = 4.5$ and standard deviation $\sigma = 1.2$.

a. Use the 68-95-99.7 rule and appropriate diagrams to find the following:

i. The value of a such that $\Pr(2.1 < T < a) = 0.95$. Give your answer correct to one decimal place. 1 mark

ii. The approximate value of $\Pr(2.1 < T < 5.7)$. Give your answer correct to three decimal places. 1 mark

iii. The approximate value of $\Pr(T < 3.3 \mid T < 4.5)$. Give your answer correct to two decimal places. 2 marks

b. Find, correct to four decimal places, the probability that on a particular day:

i. ArSchmidties spends less than 6 hours brooding by the sea. 1 mark

ii. ArSchmidties spends between 4 and 6 hours brooding by the sea. 1 mark

iii. ArSchmidties spends less than 6 hours brooding by the sea, given that he spends more than 4 hours brooding by the sea. 1 mark

c. It is known that when ArSchmidties is brooding, his thoughts turn to Patrolambous some of the time. The amount of time, P hours, that ArSchmidties spends thinking about Patrolambous follows a normal distribution with mean μ_P and standard deviation σ_P.

It is known that $\Pr(P > 2) = 0.9088$ and $\Pr(P < 3.5) = 0.8783$.

i. Find, correct to one decimal place, the values of μ_P and σ_P. 2 marks

ii. Use the standard normal distribution to express $\Pr(P > 2) = 0.9088$ and $\Pr(P < 3.5) = 0.8783$ as two simultaneous equations in terms of μ_P and σ_P.
Give all approximate values correct to four decimal places. 2 marks

Enraged by Patrolambous' death, ArSchmidties seeks out and kills Hectorcic and then ties Hectorcic's body to his chariot and drags it back to the Mathematical camp.

He lashes Hectorcic's body—miraculously healed by the Gods—to his chariot and drives around Patrolambous's burial mound, hoping to finally satisfy his grief.

Question 2 (9 marks)

The number of laps, *N*, that ArSchmidties drives around the burial mound of Patrolambous each night is a discrete random variable with probability distribution shown below.

N	1	2	3	4	5
$\Pr(N = n)$	0.05	a^2	$a^2 + 0.05$	$a + 0.07$	b

It is known that the expected number of laps is $E(N) = 3.88$.

a. **i.** State in terms of *a* and *b* two simplified simultaneous equations that could be used to find the values of *a* and *b*. 2 marks

ii. Hence write a simplified quadratic equation in terms of *a* only that could be used to find the value of *a*. 2 marks

iii. Find the values of *a* and *b*. 3 marks

b. Find the probability that N lies within one standard deviation of the mean. 2 marks

Meanwhile, within the walls of Tau, King PriaMay is mourning for his son Hectorcic. The loss of his son has brought home to PriaMay the terrible fate that all of Tau is facing. Even as PriaMay thinks about the futility of his life and reign, however, he senses a presence in the room. He turns and sees the goddess Iristina C, who tells him that the deaths of PriaMay's children and the likely fall of Tau itself are not the "mockery" of the gods or fate, but in part the result of "chance""

Iristina C then disappears, but leaves him with the challenging problem involving "chance" below.

Question 3 (6 marks)

A biased coin is tossed three times. The probability of obtaining a head with this coin is p.

a. Find, in terms of p, the probability of obtaining:

i. Three tails. 1 mark

ii. Two heads and one tail. 1 mark

b. **Hence** find the probability of two heads and one tail, given that at least one head was obtained. Give your answer as a single fraction in terms of p. 2 marks

c. Suppose that the probability of getting three heads is equal to the probability of getting two heads and one tail. Find the value(s) of p. 2 marks

PriaMay solves the problem within minutes and, excited, goes to find his wife HecGrainger.

PriaMay reveals his plan, to take a cartload of treasure to ArSchmidties as ransom for Hectorcic's body. HecGrainger is appalled by her husband's suggestion and asks PriaMay to postpone making any decisions until after he has spoken to his children and advisors.

Question 4 (12 marks)

PriaMay gathers six of his sons (including his youngest son) and ten of his advisors (including his senior advisor) together.

a. Express all answers using factorial notation.

Before these 16 people meet with PriaMay they must form an orderly queue and enter PriaMay's chambers in a single line. Find how many different ways this line can be formed with:

i. No restriction. 1 mark

ii. All of the sons occupying the first six places. 1 mark

iii. An advisor in the first place and an advisor in the last place. 1 mark

iv. Either his youngest son or his senior advisor in the first place and all of the sons together. 2 marks

b. Before PriaMay leaves, he needs to organise a committee to govern in his absence. He decides that the committee will consist of five members from the above group of 16.

Find the probability that this committee of five will be formed with:

i. The senior advisor on the committee. 1 mark

ii. More advisors than sons on the committee. 2 marks

iii. Either his youngest son or his senior advisor but not both on the committee. 2 marks

iv. More sons than advisors on the committee, given that the senior advisor is on the committee. 2 marks

PriaMay duly explains his plan to his sons and counsellors, who echo HecGrainger's concerns, arguing that PriaMay has an obligation to remain aloof and awe-inspiring as a king. PriaMay, however, again reiterates that he is a man as well as a king, and subject to pain and death like everyone else. That being the case, he wants to do something "new and unheard of" before meeting his fate.

Seeing that it is useless to argue, PriaMay's sons begin to assemble the ransom and find a cart and driver. They return with the storyteller, Bohniax, and his mule Finchy. Despite feeling somewhat overwhelmed, Bohniax agrees to take PriaMay to the Mathematics camp, and later that afternoon, the citizens of Tau watch in confusion as the two men drive out of the city.

Question 5 (9 marks)

PriaMay has the choice of two roads to take as he leaves the city. One road heads east and the other road heads west.

a. The probability of PriaMay heading east and having a safe journey is $4k^2$. The probability of PriaMay heading west and **not** having a safe journey is k.

If the probability that PriaMay chooses the road heading east is $\frac{1}{2}$:

i. Find an expression in terms of k for the probability that PriaMay has a safe journey. 2 marks

ii. Hence find the value(s) of k such that the probability of a safe journey is greater than or equal to $\frac{7}{8}$. Give your answer correct to four decimal places. 3 marks

b. The probability that PriaMay has an unsafe journey given that he takes the east road is $\frac{1}{2}$.

The probability that PriaMay takes the west road given that he has an unsafe journey is $\frac{3}{5}$.

Let q be the probability that PriaMay chooses the east road. Find the following in terms of q:

i. The probability that PriaMay makes a safe journey. Give your answer as a single fraction in simplest form. 2 marks

ii. The probability that PriaMay takes the east road, given that he makes a safe journey. Give your answer as a single fraction in simplest form. 2 marks

With evening falling and Bohniax and PriaMay stop to rest. Bohniax pities PriaMay and encourages him to join him on the banks of the river Scamander, where they dip their feet in the water and eat corkillbread and griddlecakes.

Question 6 (16 marks)

Bohniax is a superb but inconsistent cook of corkillbread. PriaMay rates this bread on a 'Corkillicious' continuous scale from 0 to 6. These ratings can be modelled by a continuous random variable with probability density function given by

$$c(x) = \begin{cases} \frac{2}{21}x, & 0 \le x \le k \\ \frac{2}{15}(6-x), & k < x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

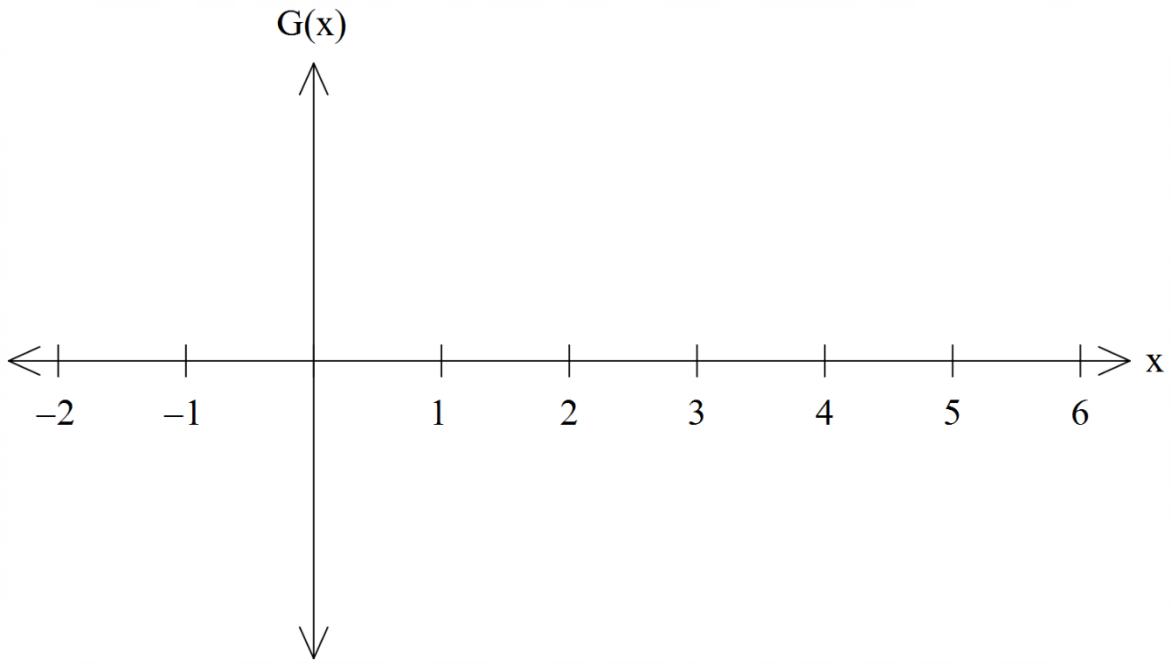
where $0 < k < 6$ is a real number.

a. Find the value of k. 2 marks

Bohniax is also an inconsistent cook of griddlecakes. PriaMay rates these cakes on a 'Griddlicious' continuous scale from 1 to 4. These ratings can be modelled by the continuous random variable, G, with probability density function given by

$$G(x) = \begin{cases} 0.2x - 0.2, & 1 \leq x \leq 2 \\ \frac{3}{16}(x-2)^2 + 0.2, & 2 < x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

b. Sketch the graph of $G(x)$ on the set of axes given below. Label all 'corners' and endpoints with their coordinates. 3 marks



c. Find, correct to **three** decimal places, the median value of G. 2 marks

d. Find, correct to **four** decimal places, the probability that a randomly selected griddlecake is rated with a score greater than its mean. 2 marks

e. Any griddlecake with a rating greater than 3.9 is rated as ‘Schmelicious’.
Find, correct to **four** decimal places, the probability that a randomly selected griddlecake is rated ‘Schmelicious’. 2 marks

f. The only rating higher than ‘Schmelicious’ is ‘Nectar of the Gods’. The probability of any griddlecake cooked being rated ‘Nectar of the Gods’ is 0.042. Bohniax cooks 20 griddlecakes.

i. Find, correct to **two** decimal places, the expected number of griddlecakes that are rated ‘Nectar of the Gods’. 1 mark

ii. Find, correct to **four** decimal places, the probability that at least three of these griddlecakes are rated ‘Nectar of the Gods’. 2 marks

PriaMay was hoping to have at least three ‘Nectar of the Gods’.

iii. Find the minimum number of griddlecakes that Bohniax must cook so that the probability that PriaMay will have at least three griddlecakes that are rated ‘Nectar of the Gods’ is greater than 0.98. 2 marks

As Bohniax chatters about the algorithms he uses to make the cakes, Priam finds himself intrigued and charmed by the "unnecessary" details of the world around him. He asks Bohniax to tell him some more stories.

Question 7 (3 marks)

Bohniax tells many stories. Some are enjoyed by PriaMay and Finchy and some are not. The probability that PriaMay enjoys a story is $\Pr(P) = p^2$ and the probability that Finchy enjoys a story is $\Pr(F) = p$. The probability that PriaMay enjoys a story is independent of the probability that Finchy enjoys a story.

Find $\Pr(F \cup P')$ in terms of p.

As the pair get underway again, they come across a young man who appears to be a Mathematical soldier, but who is in fact the god Vernies. He tells PriaMay that ArSchmidties has sent him as an escort, and though both PriaMay and Bohniax are somewhat wary of his claims and cocky demeanour, they accept his help. After the trio ford the river, PriaMay realizes who Vernies is, and the god confirms he has come to guide them to ArSchmidties's hut.

Question 8 (6 marks)

While waiting for Vernies guidance, PriaMay and Bohniax practise their archery by attempting to hit a target.

PriaMay can successfully hit the target with a probability of $\frac{6}{7}$. He shoots at the target 28 times.

Bohniax shoots at the target many times. He finds that the mean number of times he hit the target is 25 and the standard deviation of the number of times he hit the target is $\frac{5\sqrt{6}}{6}$. For both archers, the outcome of each attempt at hitting the target is independent of any other attempt.

a. Find the probability that Bohniax can successfully hit the target. 3 marks

b. Find the number times that Bohniax shoots at the target. 1 mark

c. Find, correct to four decimal places, the probability that PriaMay **and** Bohniax both **missed** the target no more than three times. 2 marks

Meanwhile, ArSchmidties is sitting in his hut. Sensing the presence of a god, ArSchmidties turns in the hopes of seeing Patrolambous's ghost. Instead, however, he sees PriaMay whom he initially mistakes for his father and falls on his knees before the visitor. PriaMay is disconcerted by this but explains who he is and why he has come.

Over a meal, the two men agree to truce for Hectorcic's funeral. Finally, PriaMay prepares to leave, and ArSchmidties tells him to call on him for help when Tau falls. PriaMay wonders aloud whether ArSchmidties will himself be alive by then, and the two share a kind of grim joke about their ultimate fates.

Question 9 (6 marks)

When planning the funeral, several ideas are discussed. It is found that the length of time, L minutes, for the funeral is a continuous random variable with probability density function given by

$$L(x) = \begin{cases} \dfrac{x}{67500} + \dfrac{1}{1500} & 30 \le x \le 180 \\ \dfrac{\pi}{360}\cos\left(\dfrac{\pi}{240}(x-180)\right) & 180 < x \le 300 \\ 0 & \text{otherwise} \end{cases}$$

a. Find the probability that the funeral lasts for less than 2 hours. 2 marks

b. Find the value of c such that $\Pr(L > c \mid L < 120) = \dfrac{1}{2}$. 2 marks

c. Find the value of a such that $\Pr(L > a) = 3\Pr(L < a)$. 2 marks

As PriaMay and Bohniax return to Tau, they pass burial mounds and a burned village, stopping only once so that PriaMay can weep in private over his son's body. Despite his grief, however, PriaMay feels rejuvenated by the journey and what he has accomplished. Back in the Mathematics camp, ArSchmidties likewise feels that a burden has been lifted from him.

PriaMay and Bohniax continue to make their way toward the city. Bohniax thinks about returning to his family, anticipating the stories he will be able to tell. Bohniax, has a reputation as a teller of tall-tales, so his grandchildren and great-grandchildren do not lend much credence to his story about conveying PriaMay to ArSchmidties's hut. Instead, they talk about how he once had an extraordinarily beautiful mule named Finchy.

END OF QUESTION AND ANSWER BOOK