

STUDENT NAME:

## MATHEMATICAL METHODS UNIT 4

### SAC 3 (Tech-Active)

# The Octonauts and the search for



## Mathlantis



Reading Time: 15 minutes

Writing time: 120 minutes

### QUESTION AND ANSWER BOOK

<i>Number of Questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	71
		Total 71

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one laptop with Mathematica and any number of Mathematica files.
- Students are NOT permitted to bring into the examination: blank sheets of paper and/or correction fluid/tape.

#### Materials supplied

- Question and answer book of 24 pages.
- Formula sheet

#### Instructions

- Write your name in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.**

**This page is blank, as always**

*Note: Sections between these symbols are story only and are not needed to answer the questions*



*Our story begins in the Octopod with three of our Octonauts poring over a map.*

*Captain Rowen Maynacles, Lt. John Kwazimond and Luke Peshoni are studying a map that Maynacles found whilst looking through his old school assessments.*

*“Flappity flippers” exclaimed Peshoni, “This looks like a map showing some clues to the location of the lost institution of Mathlantis!”*

*“Mathlantis? What’s Mathlantis” asked Kwazimond?*

*“You know! It was the sea’s first specialist institution for the sciences, mathematics and associated technologies” replied Maynacles.*

*“Shiver me whiskers” exclaimed Kwazimond. “I hope it wasn’t administered by those dopey boneheads from SEA-CAR.”*

*“I believe that it was,” replied Maynacles “and that may be why it disappeared without a trace . . . until now!”*

***Octonauts to the launchbay!!! ”***

*The Octonauts race to the launchbay where they find their chief engineer Nadine Pheaks preparing ten of their specialist vehicles, known as SUPs, to take on their exploration. They’ll be ready faster than you can say “buncha munchy, crunchy carrots” exclaims Pheaks.*

*SUP-A, SUP-B all the way through to SUP-J are all ready and waiting in the launchbay.*



**Instructions**

Answer **all** questions in the spaces provided

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1** (10 marks)

There are 10 vehicles, named SUP-A, SUP-B, SUP-C, SUP-D, SUP-E, SUP-F, SUP-G, SUP-H, SUP-I and SUP-J waiting for the Octonauts in the launch bay.

Nadine Pheaks has lined them up in a single row.

“Remember,” says Pheaks .... “SUP-A, SUP-E and SUP-I are vowel vehicles and the others are consonant vehicles.” Pheaks is a linguistics legend!

- a.** Find the number of ways the 10 SUP-vehicles can be arranged. 1 mark

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- b.** Find the probability that:

- i.** SUP-A, SUP-B and SUP-C are the first three vehicles in the row and in that order. 1 mark

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- ii.** The vowel vehicles SUP-A, SUP-E and SUP-I are all next to each other. 2 marks

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- iii.** All of the consonant vehicles are **not** together. 2 marks

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The Octonauts decide that they only need five of these ten vehicles for their mission.

- c.** In how many ways can they select five of these vehicles if:

- i.** There is no other restriction. 1 mark

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- ii.** They must take SUP-B. 1 mark

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- iii.** They take either SUP-B or SUP-J but not both. 2 marks

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*The Octonauts have been briefed by Maynacles and select their appropriate SUPs for their adventure.*

*“Explore, Rescue, Protect!!”*

*According to the map, the Octonauts must first pass through a maze of sea caves known as the Mathematicaves. Legend has it that many explorers have been lost wandering in endless loops in the deep recesses of the Mathematicaves, unaware that they needed to perform a ritual known as quitting the kernel to be able to exit.*



**Question 2** (8 marks)

The Octonauts find their way to the Mathematicaves. There are three separate entrances,  $A$ ,  $B$  and  $C$ , leading to pathways through the Mathematicaves.

The Octonauts do not know that the probability of getting through the Mathematicaves depends on the chosen entrance. These probabilities are shown in the table below.

Entrance	$A$	$B$	$C$
Probability of getting through the Mathematicaves	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{3}$

- a. i.** Kerzimon, trusting his instincts, (what could possibly go wrong?) chooses an entrance at random. Show that the probability that he gets through the Mathematicaves is  $\frac{13}{36}$ . 2 marks

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- ii.** Given that Kerzimonnd gets through the Mathematicaves, calculate the probability that he chose entrance  $A$ . 2 marks

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Dr. Peter Rossitani has a more measured approach to passing through the Mathematicaves.

The probability that Dr. Rossitani chooses entrance  $A$  is  $p$ . The probability that he chooses entrance  $C$  is twice the probability that he chooses entrance  $B$ .

- b. i.** Find, in terms of  $p$ , the probability that Dr. Rossitani gets through the Mathematicaves. 2 marks

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- ii.** Hence find the value(s) of  $p$  for which the probability of Dr. Rossitani getting through the Mathematicaves is greater than the probability of Kerzimonnd getting through the Mathematicaves. 2 marks

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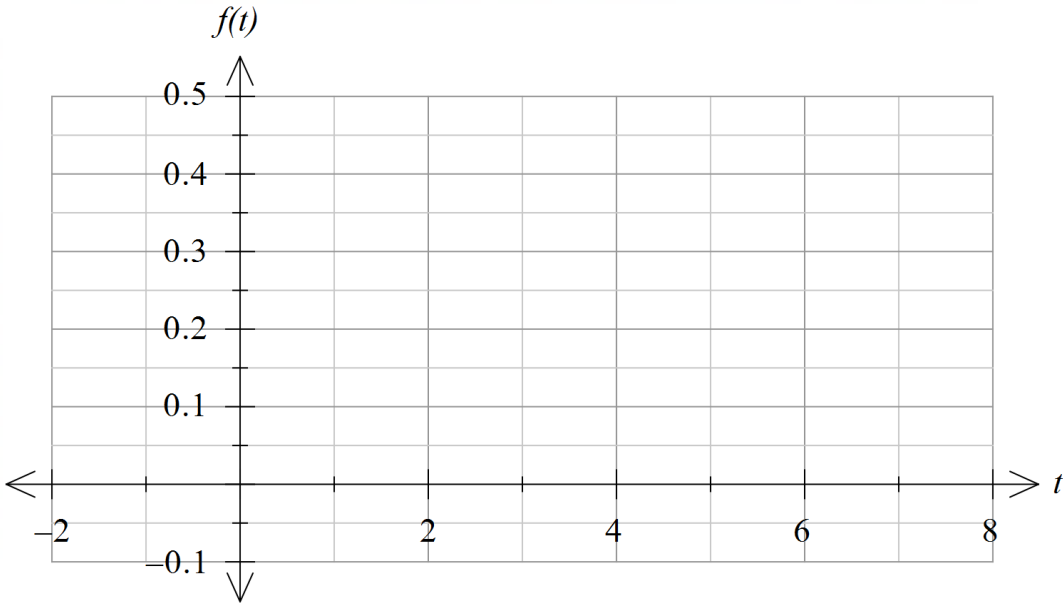
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Use the value  $h = -28$  for parts **b**, **c**, and **d**. below.

- b.** Sketch the graph of  $y = f(t)$  on the axes provided below, labelling all endpoints. 3 marks



- c.** Find  $\Pr(2 \leq T \leq 4)$ . 1 mark

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- d.** Find, correct to four decimal places, the value of  $a$  such that  $\Pr(T \geq a) = 0.8$ . 2 marks

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*Back to the Octonauts that have passed through the Mathematicaves!*

*On their exit from the Mathematicaves, the Octonauts bump into their old pirate friend Calico Finch, with his team of pirates, who is are on the search for Mathlantis and coincidentally have just navigated their way through the Mathematicaves. (What’s the chance of that?? Low probability, I’m thinking.)*

*His research indicates that the next part of their journey will take them through a trench that is infested with stinging jellyfish and slippery binomy eels. At the other side of the trench is a treasure chest with further information and clues to the location of Mathlantis.*

*“Any volunteers to investigate the trench?” asked Maynacles.*

*Already on it Captain!” responded Kristina Dashkovska, one of the bravest of all Octonauts.*

*Dashkovska and Calico Finch decide to lead the way through the trench.*



**Question 4** (5 marks)

It is known that the number of stings,  $N$ , received from the jellyfish when passing through the trench has the probability distribution shown below.

$n$	0	1	2	3	4
$\Pr(N = n)$	$\frac{3c}{2}$	$d + 0.01$	$d - c$	$d - \frac{5c}{2}$	$\frac{c}{2}$

The expected value of  $N$  is 1.81.

- a.** Write in simplified form two equations that could be used to find the values of  $c$  and  $d$ . 2 marks

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The values of  $c$  and  $d$  are  $c = 0.04$  and  $d = 0.35$ .

- b.** Find, correct to two decimal places, the standard deviation of  $N$ . 1 mark

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- c.** Given that Calico Finch gets at least two stings, find the probability that he got four stings. 2 marks

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*All of the Octonauts get safely through the trench!*

*Unfortunately, Calico Finch sustains the maximum number of stings and needs some medical treatment from Peshoni.*



**Question 5** (7 marks)

Peshoni bandages the wounds on Calico Finch the Pirate and then decides to administer a painkiller called Peshoni’s Pain-away.

The amount of Peshoni’s Pain-away required to relieve all pain in a pirate is a normally distributed random variable with a mean of 30 milligrams and a standard deviation of 8 milligrams.

If too much of Peshoni’s Pain-away is administered, it can result in a condition called chronic grumpiness. For a pirate, the amount of Peshoni’s Pain-away required to trigger chronic grumpiness is a normally distributed random variable with a mean of 70 milligrams and a standard deviation of 18 milligrams.

- a. Calico Finch is given a dose that will relieve all pain in 90% of pirates. Find, correct to four decimal places, the probability that he gets chronic grumpiness. 2 marks

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Calico Finch is super grumpy. He is grumpy about his pain, the stings, Mathlantis, the SUPs, and grumpy about his team of pirates and their lack of knowledge about the normal distribution curves. Calico Finch decides to give his team of pirates a surprise test of their understanding.

Let  $X$  be a normally distributed random variable with mean  $\mu = 30$  and standard deviation  $\sigma = 8$ . Let  $Z$  be the standard normal random variable.

- b.** Find the value of  $a$  such that  $\Pr(X < 46) = \Pr(Z > a)$ . 1 mark

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- c.** Using appropriate diagrams of the normal and standard normal distribution curves, find the value of  $b$  such that  $\Pr(22 < X < 34) = \Pr(b < Z < 1)$ . 2 marks

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Calico Finch is arguing with a fellow pirate about their over-reliance on technology and that they should use the “feel and flow” of the maths and graphs instead.

- d.** Use the 68–95–99.7% rule to find the value of the integer  $c$  such that  $1 - \Pr(X < c) \approx 0.84$ . 2 marks

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Once Calico Finch, has made a full recovery (although he is still a bit grumpy!), the Octonauts continue with their search. Katherine Fitznip and Dr Phil Schmidtington find the treasure chest of which Calico Finch had spoken, which promises to reveal the location of Mathlantis. The treasure chest is locked with a message stating that the locks can only be opened by correctly answering the probability questions below. “Jumping Jellyfish!”, exclaims Schmidtington. “Let me look at those, I love probability problems.”

“Me too!” squeaks an excited Fitznip.



**Question 6** (5 marks)

Two events  $A$  and  $B$  are such that  $\Pr(A|B) = \frac{2}{3}$  and  $\Pr(B|A) = \frac{1}{4}$ . Let  $\Pr(A \cap B) = p$ .

a. Find  $\Pr(A)$  in terms of  $p$ .

1 mark

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b. Find  $\Pr(A' \cap B')$  in terms of  $p$ .

2 marks

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c. Find all possible values of  $p$  if  $\Pr(A \cup B) \leq \frac{1}{2}$ .

2 marks

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*Schmidtington and Fitznip have no problem solving these problems and the locks on treasure chest spring open and an apparition of two vegimals Ambarat and Dulapeno appear before them with the message:*



*Fitznip passes this message on to Schmidtington who is fluent in Vegimalese and then with a “Cheepa, Cheepa,” Fitznip returns to her SUP to bake up some treats.*

*“Jumping Jellyfish!” ,again exclaims Schmidtington*

*(I must extend my exclamation vocabulary he thinks to himself!)*

*The message reads” In order to obtain the final instructions to locate Mathlantis, you must correctly answer at least three of the riddles of the clownfish.*

*Maynacles is the king of riddles. He plays an online game of “Riddle me this!” in the Octopod most evenings whilst playing his accordion.*



**Question 7** (6 marks)

The number of riddles Maynacles answers correctly in the incredibly difficult “*Riddle me this!*” game follows a binomial distribution with mean  $\mu = 6$  and variance  $\sigma^2 = \frac{21}{4}$ .

In a game of “*Riddle me this!*”:

- a. Show that the number of questions that are asked is 48 and that the probability of a successful answer to each riddle is  $\frac{1}{8}$ . 2 marks

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- b. Find, correct to four decimal places, the probability that Maynacles answers more than three questions correctly. 1 mark

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- c. Find the probability that Maynacles answers only the first ten questions correctly.

Express your answer in the form  $\frac{a^m}{b^n}$  where  $a, b, m, n \in \mathbb{Z}^+$ . 1 mark

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*Maynacles approaches the clownfish and says “ Hit me with your riddles, clownfish! My regular participation and success in “Riddle me this!” has prepared me for this very moment.*

*“Riddle me this! is for numbskulls” laughs the clownfish. “My riddles are much more challenging, and you must answer at least three riddles correctly before you can continue with your search.*



The number of correct answers to the clown fish’s riddles follows a binomial distribution with the probability of answering a single riddle correctly given by  $p = \frac{1}{12}$ . To continue with the search, Maynacles must answer at least three riddles correctly.

- d.** Find the minimum number of riddles that need to be asked so that the probability that Maynacles answers at least three riddles correctly is greater than 0.9.

2 marks

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*Maynacles is a known big game performer and has no trouble answering three riddles.*

*The clownfish explains that by correctly answering three of his riddles he can now lift the invisibility shield that protects Mathlantis and he is able to direct the Octonauts towards their goal.*

*“The shield will only remain lifted for 496 minutes,” states the clownfish, “so you will need to find Mathlantis and enter its gates before then.”*

*“Perfect!” exclaims Maynacles, knowing that 496 is a perfect number.*



**Question 8** (3 marks)

The Octonauts leave the clown fish's lair and continue towards Mathlantis, aware that they only have 496 minutes to get to Mathlantis before it becomes invisible again.

It is known that the time taken,  $T$ , to travel to Mathlantis from the clown fish's lair follows a normal distribution with mean  $\mu_T$  and standard deviation  $\sigma_T$ .

It is also known that  $\Pr(T > 455) = 0.9546$  and  $\Pr(T > 540) = 0.2993$ .

- a. Find, correct to the nearest integer, the values of  $\mu_T$  and  $\sigma_T$ .

2 marks

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- b. Hence find, correct to four decimal places, the probability that the Octonauts arrive at Mathlantis before it becomes invisible again.

1 mark

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*Our courageous Octonauts arrive at the imposing gate of Mathlantis with time to spare.*

*The impressive sign out the front reads:*

***MATHLANTIS:***

***The sea's first specialist institution for the sciences, mathematics and associated technologies.***

*They are greeted at the Gates by the famed principal Peter Corkfish who was one of the key architects in setting up Mathlantis.*

*He explains that after a brilliant start to its operations, many students became complacent, not listening to their instructors, preferring instead to outsource their learnings to private, but unauthorised 'educators'. Despite this glitch, other institutions became jealous of the culture of Mathlantis. Word soon filtered through to the authorities at SEA-CAR, who in their wisdom, decided to conduct an audit of Mathlantis and its practices.*

*The students were unable to impress the auditor, so Mathlantis, along with Corkfish was banished to the remote depths of the ocean and shielded by an invisibility spell.*

*"The only way to return Mathlantis to its former glory is for me to demonstrate that we are worthy and are able to demonstrate excellent mathematical learnings." explained Corkfish*

*If we are able to answer the two following questions correctly, we have been promised that Mathlantis can reclaim its rightful place in the great education continuum.*

**Question 9** (10 marks)

a. A random variable  $W$  has the probability density function

$$w(x) = \begin{cases} 0.1 & 0 \leq x \leq k \\ \sin\left(\frac{2x-5}{4}\right) & k < x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

Find, correct to four decimal places, the value(s) of  $k$ .

2 marks

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b. The time  $t$  hours spent on a tricky problem is a continuous random variable,  $T$ , with probability density function

$$f(t) = \begin{cases} \sqrt{2} \cos(t) & 0 \leq t \leq \frac{\pi}{4} \\ 0 & \text{elsewhere} \end{cases}$$

i. State the derivative of  $t \sin(t)$ .

1 mark

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The Octonauts ponder the above problem. They know that the maximum time that can be spent on the problem is  $\frac{\pi}{4}$  hours. They would prefer to know, once they start the problem, how much time they have remaining.

Let the function  $g(t) = f(mt + n)$  be a transformation of the function  $f(t)$ , where  $t$  is the time remaining to solve the problem and  $m$  and  $n$  are real constants.

iv. Find the values of  $m$  and  $n$ .

2 marks

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*Corkfish and the Octonauts believe that they have been able to solve these problems correctly.*

*“Now it is time to send our solutions to SEA-CAR so that we can be restored to our former glory”  
says Corkfish.*

*Whilst the team are deciding who should take the solutions to SEA-CAR, Kwazimond  
jumps into SUP-B and races off with the solutions.*

*“I’m the fastest, especially when I am in SUP-B. I’ll have the solutions in SEA-CAR’s hands before  
these guys have finished their meeting” he thinks to himself . . . . .*