


Name: _____ Teacher: _____

	METHODIST LADIES' COLLEGE MATHEMATICAL METHODS — UNIT 4 Probability and Statistics Problem Solving Task SAC 3, 2021
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Component 1 – Monday 30th August

Directions to Students

- You have 60 minutes of Writing Time to complete each component. NO Reading Time.
- Each component will be available on CANVAS.
- Answer **all** questions in the spaces provided.
- Unless otherwise specified, an **exact** answer is required to answer a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Write your **name and teacher name** in the space provided.
- All written responses must be in English.
- RESET your CAS at the end of the component and show your teacher.
- SCAN and upload every page, submit on CANVAS.

Materials Allowed

- 1 CAS calculator
- 1 bound reference
- VCAA Mathematical Methods formula sheet.

NOTE: The mark you receive for this SAC is subject to moderation following your exam results at the end of the year.

Outcome 1	Outcome 2	Outcome 3
16 marks	18 marks	11 marks

The Hawthorn Chocolate Company (HCC) has just celebrated its 5th Birthday. 5 years ago, they opened a store on Cotham Road and an online store. They sell speciality chocolates in a variety of flavours and sizes.

Question 1

Over the years, a large number of customers have come into the shop to browse and purchase chocolates. The time (T minutes) a customer spends in the store is normally distributed with a mean of 17 minutes and a standard deviation of 9 minutes.

a) i) Find the probability that a randomly selected customer spends less than 12 minutes in store. Give your answer correct to four decimal places.

ii) The probability that a customer will spend more than w minutes in the store is equal to 0.83. Find w , correct to one decimal place.

iii) The mean amount of time a customer spends instore was found to be incorrect.

By exploring different mean times, discuss the change in the probability of a randomly selected customer spending less than 12 minutes in store.

Philip is a salesperson in the store. The probability that he spends more than 2 minutes helping a customer is $\frac{5}{8}$.

b) i) Find the probability that Philip spends more than 2 minutes with at least 4 of the 7 customers he helped during his shift. Give your answer correct to three decimal places.

ii) Find the probability that Philip spends more than 2 minutes helping the first 5 customers he serves during another shift. Give your answer correct to three decimal places.

iii) What **one** assumption has been made when calculating the probability that Philip spends more than 2 minutes helping the first 5 customers he serves during another shift?

The store had to shut their doors during lockdown, but customers were able to make purchases online. The HCC were able to view how much time was spent on their website.

c) The time (S seconds) a customer spends on the online store is normally distributed with a mean of 320 seconds and a standard deviation of d seconds.

The probability that an online customer spends at least 390 seconds is 0.2455. Find the value of d , correct to 2 decimal places.

d) Describe **two** differences between the amount of time customers spend in store compared to shopping online. Explain what these differences mean within the context of shopping in store and online. (Use the in store mean shopping time as 17 minutes).

Question 2

Once the HCC was able to re-open after lockdown, they became interested in the types of customers who were visiting the store so that they could improve their advertising. They asked a group of customers their age (years) and how far they travelled (km) to make purchases in store.

Let A represent customers who are older than 25 years old.

Let B represent customers who have not travelled further than 5km.

a) Complete the following probability table.

	B	B'	
A	0.6		
A'		0.22	0.32

b) Explain what the value 0.32 in the table represents in the context of customers visiting the HCC store?

c) What is the probability that a customer selected at random is older than 25 and travels further than 5km or a customer who is 25 or younger and does not travel further than 5km?

d) If a customer is selected at random, find the probability that they have not travelled more than 5km given that they are older than 25?

e) Show that events A and B are not independent. Explain what this means in the context of customers visiting the HCC store.

Question 3

As part of HCC's 5th birthday celebrations, a mixed bag of chocolate coated fruit and nuts are on sale. There are unequal pieces of chocolate covered cashew nuts, almonds, sultanas and apricots in the mixed bag. The fruit and nuts are covered in milk chocolate or white chocolate.

The HCC try to ensure that 18% of the chocolates are sultanas which are covered in either milk or white chocolate. One mixed bag was opened, and 6 pieces of chocolates were chosen at random.

a) What is the probability that at least 2 of these chocolates have sultanas in the centre? Give your answer correct to four decimal places.

34% of the pieces of chocolates in the mixed bag are covered in white chocolate.

b) What is the probability that 1 out of the 6 randomly selected chocolates is a white chocolate covered sultana? Give your answer correct to four decimal places.

c) What is the minimum number of pieces of chocolate that need to be sampled from the bag to ensure that the probability of choosing least two milk chocolate covered pieces is more than 0.65?

Question 4

As an incentive to bring more local students into the store, the HCC started a bonus give away. If a student shows their student ID card they can reach into a barrel and select a combination of green or red covered chocolate hearts.

Let X be number of green covered chocolate hearts taken out of the barrel by the student.

The discrete probability distribution of X is given by the formula $Pr(X = x) = \frac{1}{42}(5x + 3)$ for $0 \leq x \leq n$, where $n \in \mathbb{Z}^+$.

a) Use the probability distribution function to fill in this table and to find the value of n .

x	
$Pr(X = x)$	

b) Explain why this formula can represent a discrete probability distribution.

c) If a student was selected at random, find the probability that they picked out an odd number of green chocolate hearts?

d) Using part a) show that, $E(X^2) \neq [E(X)]^2$.

e) What is the 95% confidence interval for the number of green chocolate hearts selected from the barrel.

f) One Friday afternoon, 84 local students came into the store and showed their ID. How many students would be expected to select less than 2 green chocolate hearts?

END of COMPONENT 1