Name: _____ **Teacher:** _____

MLC	**METHODIST LADIES' COLLEGE** **MATHEMATICAL METHODS — UNIT 4** **Probability and Statistics Problem Solving Task** **SAC 3, 2021**

Component 2 – Wednesday 1st September

Directions to Students

- You have 60 minutes of Writing Time to complete each component. NO Reading Time.
- Each component will be available on CANVAS.
- Answer **all** questions in the spaces provided.
- Unless otherwise specified, an **exact** answer is required to answer a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Write your **name and teacher name** in the space provided.
- All written responses must be in English.
- RESET your CAS at the end of the component and show your teacher.
- SCAN and upload every page, submit on CANVAS.

Materials Allowed

- 1 CAS calculator
- 1 bound reference
- VCAA Mathematical Methods formula sheet. Available to print prior to SAC.

NOTE: The mark you receive for this SAC is subject to moderation following your exam results at the end of the year.

Outcome 1	Outcome 2	Outcome 3
9 marks	20 marks	17 marks

The Hawthorn Chocolate Company (HCC) has just celebrated its 5th Birthday. 5 years ago, they opened a store on Cotham Road and an online store. They sell speciality chocolates in a variety of flavours and sizes.

Question 5

The master chocolatier, Gerald works for a certain number of hours each day depending on how much stock is required.

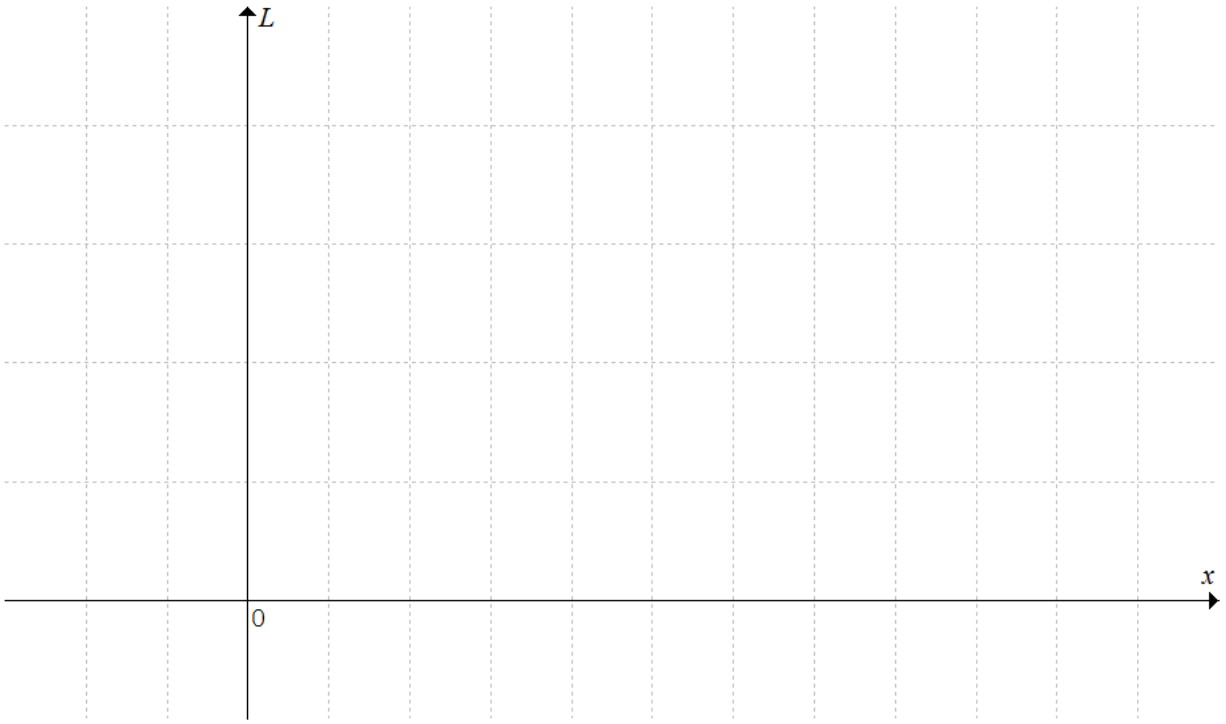
The number of hours he works $L(x)$ can be modelled by the following probability density function:

$$L(x) = \begin{cases} \frac{bx}{5} & 0 \leq x \leq 5 \\ bcos\left(\frac{\pi(x-5)}{10}\right) & 5 < x \leq 10 \\ 0 & otherwise \end{cases}$$

a) Show that $b = \frac{2\pi}{5(\pi+4)}$

b) What is the probability that Gerald works less than 6 hours per day? Give your answer correct to 3 decimal places.

c) Sketch $L(x)$ on the axes below, labelling any axis intercepts, turning points and endpoints with their co-ordinates.



d) Find the mean number of hours Gerald works per day. Give your answer correct to three decimal places.

e) Calculate the median number of hours Gerald works per day. Give your answer correct to three decimal places.

f) Gerald drives to work and parks his car in a 7 hour parking bay.

What is the probability that he will have to move his car during one of his shifts? Give your answer correct to two decimal places.

g) Gerald must have a 30 minute break for every 6 hours he works per day and a 15 minute break for every additional hour.

What is the probability that he will need at least a 1 hour break? Give your answer correct to three decimal places.

Francis is a chocolatier at a rival chocolate shop. The number of hours he works each day can be modelled using the following distribution:

$$F(x) = \begin{cases} \dfrac{4}{3(x-0.5)^2} & 1.5 \leq x \leq 4.5 \\ 0 & elsewhere \end{cases}$$

h) Identify three aspects of Francis's work hours. Justify your solutions with mathematical evidence. Comment on how Francis's work hours compare to Gerald's.

Question 6

The HCC store tracked the number of chocolate bars sold per purchase on a Saturday. The number of chocolate bars sold per purchase can be described using the following discrete probability distribution.

x	0	1	2	3
$Pr(X = x)$	t^2	p	t^2	$2t^2$

a) Given that $t + p = -0.5$, find the value of t and p.

b) Find the expected number of chocolate bars sold per purchase.

The HCC wanted to sell more chocolate bars, so they offered all customers a discount off their total purchase price. The more chocolate bars they bought, the larger discount they received.

- 1% discount applies if they buy no chocolate bars
- 4% if they buy 1 chocolate bar
- 7% if they buy 2 chocolate bars
- 10% if they buy 3 chocolate bars.

All of the customers who shopped spent exactly $50. The probability of them buying b chocolate bars is listed in the table below.

c) i) Complete the table below where Revenue (R), in dollars, is the amount received by the HCC for the purchases less the discount.

b	0	1	2	3
$Pr(B = b)$	0.07	0.13		0.35
Revenue ($R)				

ii) Calculate the expected revenue per customer.

iii) Find E$(2R + 1)$ and Var$(2R + 1)$. Give your answers correct to two decimal places.

Question 7

On the HCC's 5th Birthday, if a customer came into the store and said "Happy Birthday" to a staff member they were given a take home gift.

a) i) A sample of 95 customers, from the larger population of all customers, were asked whether they enjoyed the gift. 25 responded that they did not. Construct a 95% confidence interval for the proportion of customers who did not enjoy the gift. Give your answer correct to 3 decimal places.

ii) Find the margin of error for this 95% confidence interval. Give your answer correct to 3 decimal places.

The manager was interested to see what would happen to the size of the 95% confidence interval if the sample size changed.

b) In each of the following, the sample proportion remains unchanged.

i) If the sample size is multiplied by 3 what effect does that have on the size of the confidence interval?

ii) If the sample size is multiplied by $\frac{1}{3}$ what effect does that have on the size of the confidence interval?

iii) If the sample size is multiplied by r, where $r \in Z^+$, what effect does that have on the size of the confidence interval, in terms of r.

When planning the take home gift, the manager was 99% sure that 70% to 80% of customers would enjoy the gift.

c) What sample size and number customers who enjoyed the gift would be required for the manager make this level of confidence?

Question 8

After Easter 2021, the HCC sold broken pieces of chocolate easter eggs in bags of three sizes - small, medium, and large. The weight of the broken chocolate easter eggs determine which size bag was used.

a) The weights of the broken easter egg filled bags are normally distributed.

- 19% of the broken easter eggs were put into large bags and have a weight greater than 250 grams
- 21% are distributed into the small bags which weigh less than 147 grams

Find the mean and the standard deviation weight of the bags of broken chocolate easter eggs. Give your answers correct to two decimal places.

The manager of the HCC found another box of broken chocolate easter eggs in the storeroom. The weights of the broken chocolate easter eggs bags in the box follow a different distribution.

e) Design a suitable probability density function $B(x)$, that meets the following criteria.

- It is a continuous piecewise function, with at least two functions.
- The mean weight is from 160 and 200 grams.
- The maximum weight of a bag is 270 grams.

State any assumptions you are making when designing this PDF.

END of COMPONENT 2