**MARKING SCHEME 2015 VCE Mathematical Methods Unit 4 SAC 5 Analysis Task Probability Chapters 14 to 18**

• Deduct a maximum of one mark overall for rounding error

• Deduct a maximum of one mark overall for units of measurement error

• Deduct a maximum of one mark overall for missing *dx*

• All other marks are deducted per question

**Question 1** (19 marks) **a.** Show that *k* = 0.23. 1 mark

0.3 + 0.4 + *k*

+ 0.05 + 0.02 =

1

(M1) *k*

=

0.23

**b.** For the number of spots on the modified Dalmation dog:

**i.** find the mean 2 marks

*E* ( *X*

) 2 0.3 3 0.4 4 0.23 5 0.05 6 0.02 *E* ( *X*

) 3.09

= × + × + × + × + × =

(M1) (A1)

**ii.** find the median 1 mark

3

**iii.** show that the variance is 0.9019 1 mark

*Var* ( *X* ) = *E* ( *X* 2

) −

[ *E* ( *X*

) ]2 *Var* ( *X*

) = 2 2 × 0.3 + 3 2 × 0.4 + 4 2 × 0.23 + 5 2 × 0.05 + 6 2 × 0.02 −

3.09

2

*Var* ( *X*

) =

0.9019

(M1)

**iv.** find *Var*(3*X* – 2) 2 marks

= 32(0.9019) (A1) = 8.1171 (A1)

**v.** find Pr( μ − 2 σ ≤ *X* ≤ μ + 2 σ ) **and** interpret your result. 3 marks

Pr( μ − 2 σ

) =

1.191 Pr( μ + 2 σ

) =

4.989 Pr(1.191 ≤ *X*

≤

4.989) = Pr(2 ≤ *X*

≤

4) =

0.93(M1) (M1) ∴

93% of the data lies within 2 standard

OR deviations of the mean

∴

93% of dogs will have between 2

(A1) and 4 spots inclusive

The large population of genetically modified dogs are kept in a secure open range enclosure.

**c.** If two dogs are randomly chosen from the enclosure, find the probability that

**i.** each dog has three spots, correct to 2 decimal places 1 mark

0.42 = 0.16

**ii.** the total number of spots on both dogs is equal to seven 2 marks

Pr(total 7) = Pr(2 and 5) + Pr(3 and 4) + Pr(4 and 3) +

Pr(5 and 2) = 2 ( 0.3 × 0.05 + 0.4 ×

0.23

) =

0.214

(M1) (A1)

**iii.** one of the dogs has two spots, given that the total number of spots on

both dogs is equal to seven, correct to 4 decimal places. 2 marks Pr(one dog 2 spots|total spots is 7)

=

Pr(one dog Pr(total 2 spots spots and total is 7)

spots is 7)

=

Pr(2 Pr(total and 5) spots + Pr(5 is and 7)

( )

2)

=2 0.3 0.214

× 0.05

=

0.1402

(M1)

(A1)

**d.** Eight modified Dalmation dogs are randomly chosen from the enclosure.

**i.** Find, correct to 4 decimal places, the probability that a dog chosen at

random from the eight dogs has two spots. 1 mark 0.1977

**ii.** Find, correct to 4 decimal places, the probability that at least four of the

dogs have two spots. 1 mark 0.1941

**e.** A random group of genetically modified Dalmation dogs manage to escape from

the enclosure. Find the smallest number of dogs in the group if the probability that at least two of the dogs in the group, having less than four spots, is greater than 0.99. 3 marks

*X* ~ *Bi* ( *n*

, 0.7) Pr( *X*2) 0.99 Pr( *X*

1) 0.01 *n* 1 (0.7)(0.3) 1

*n* 0 (0.3) 0.01 *n*

6.085 7 dogs

≥ >

}(M1) Either line

≤ ≤

   (M1) Also accept equals sign

(A1)    +       ≤ ≥∴ *n* −

*n*

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**Question 2** (10 marks) It is found that when an **unmarried** couple go and see a movie together, there is a 58% chance that they will see a romantic comedy (RomCom) if the previous movie they saw together was a RomCom, and a 46% chance that they will see an action movie if the previous movie they saw together was an action movie.

**a.** Create a transition matrix for the probabilities of an unmarried couple seeing either

a RomCom or an action movie together, given the previous movie seen together. 1 mark

*T* =    0.58 0.42 0.54 0.46   

or *T* =    0.46 0.54 0.42 0.58   

It is found that when a **married** couple go and see a movie together there is a 92% chance that they will see a RomCom if the previous movie they saw together was an action movie, and a 5% chance that they will see an action movie if the previous movie they saw together was a RomCom. A transition matrix for the probabilities of a married couple seeing either a RomCom or an action movie together given the previous movie seen together is

*T* =    0.95 0.05 0.92 0.08   

**b.** John and Susan are a **married** couple who decide to go and see a movie together. It is equally likely that they will see either a RomCom or an action movie the first time that they go to the movies together. Find the probability that the second time they go to the movies together, John and Susan will see a RomCom 2 marks Pr(RomCom second time) 0.5 0.5

Pr(RomCom second time) 0.935 0.065

0.935

= *T* ×       =       ∴

(M1) Matrix

(A1) Answer stated outside of matrix

**c.** As it happens, the first movie that John and Susan saw in 2015 was an action movie. Find:

**i.** the probability that the next three movies John and Susan see together

are RomComs. 1 mark 0.92 × 0.95 2 = 0.8303 **ii.** the probability that the fifth movie they see together in 2015 will be a

RomCom, correct to 4 decimal places. 2 marks Pr(fifth movie is RomCom) 4 0 1

Pr(fifth movie is RomCom) 0.9485 0.0515

0.9485

= *T* ×       =       ∴

(M1) Matrix

(A1) Answer stated outside of matrix

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**iii.** the steady state probability that they will see an action movie

together, giving your answer as an exact value 2 marks Steady state = 0.05 0.05 +

0.92

Steady state =

97 5 (M1) Also accept use of *Sm* and *Sn*, where *m* and *n*

are large numbers with use of ApproxFraction

(A1)

On a particular day it was found that 84 married couples saw an action movie together and 212 married couples saw a RomCom together.

**d.** Find, correct to the nearest whole number, the number of married couples expected

to see each type of movie in the long term. 2 marks RomCom = 92 97

× (84 + 212) RomCom = 281 Action = 84 + 212 −

281 Action =

15(A1) (A1)

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**Question 3** (15 marks) A social researcher carried out a lifestyle survey on a group of people. A mathematical model was then developed from the result. The time, in hours, spent each week on sport and recreation by people surveyed is a continuous random variable, *T*, with probability density function

*f* ( *t* ) =   

2 − 2

4 4 *kt* − 13 *rt e t e* + 13

116 *e* 0 for 0 ≤ *t*

≤ 8

for 8 < *t* ≤ 14.5 elsewhere

**a.** Given that *f* (*t*) is continuous at *t* = 8, show that *k* = 16 1and *r* = 8. 2 marks

∫ 0

14.5

*f* ( *t* ) *dt* = 1 or ∫ −∞

∞*f* ( *t* ) *dt*

=

1

(M1)

*k* (8) 2 *e* −

8 2

= − 13 *r e* 4 (8) +

13 116 *e* 4

(M1)

∴ *k* = 16

1 and *r*

= 8 **b.** Sketch the probability density function *f*(*t*) on the axes below. Label any

intercept(s) and turning point(s) using coordinates. 3 marks

Intercept (0, 0) (M1⁄2) Intercept (14.5, 0) (M1⁄2) Turning point    4, 1 *e*

2   (M1⁄2) 0 elsewhere shown (M1⁄2) Correct shape (M1) Total is sum of marks rounded down (M3)

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**c.** Find the median time spent on sport and recreation by this group, in hours,

correct to 2 decimal places. 2 marks ∫ 0 *m f* ( *t* ) *dt*

=

0.5

(M1)

*m*

= 5.35 hours

(A1)

**d.** Find the modal time spent on sport and recreation by this group. 1 mark

4 hours

**e.** A person is chosen at random from the group. Find the probability that the person:

**i.** spends at least 5 hours on sport and recreation, correct to four decimal

places 1 mark ∫

5 14.5 *f* ( *t* ) *dt* = 0.5438 **ii.** spends less than 8 hours on sport and recreation, correct to four decimal

places 1 mark ∫ 0 8*f* ( *t* ) *dt* = 0.7619 **iii.** spent less than 8 hours on sport and recreation, given that they spent at

least 5 hours on sport and recreation, correct to 2 decimal places. 2 marks Pr( *X* < 8 | *X* > 5) = Pr(5 Pr( < *X*

*X* 8) > 5)

Pr( *X* 8 | *X*

5) 85*f* ( *t* ) *dt* 0.5438 Pr( *X* 8 | *X*

5) 0.56 < < > =

< > =

∫ (M1)

(A1)

The social researcher decides to interview 6 people from the group to see whether or not the people spend at least 5 hours on sport and recreation each week.

**f.** Find the probability, correct to two decimal places, that three people spend at

least 5 hours on sport and recreation each week. 2 marks Pr( = 3) =    6 3    (0.5438) 3 (0.4562) 3 Pr( = 3) =

0.31 *X*or *X* ~ *Bi* ( 6, 0.5438 ) (M1)

*X*

(A1)

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**Question 4** (16 marks) Records kept by Massivechusettes Institute of Technology suggest that the distribution of the total fees paid by graduating students is normal, with mean $60000 and standard deviation $5000.

**a.** What proportion of the university’s graduates pay:

**i.** less than $55000, correct to 4 decimal places? 1 mark

0.1587

**ii.** more than $50000 but less than $74000, correct to 4 decimal places? 1 mark

0.9747

**b.** The university’s advertising manager wishes to claim that ‘90% of our graduating

students pay less than *c* in total fees. What should *c* be, correct to the nearest dollar? 1 mark $66408

Jeremy is practising shots from the free-throw line on the university basketball court. If Jeremy makes a shot, the probability he will make the next shot is 0.6. If he misses a shot, the probability he will make the next shot is 0.3. Jeremy misses his first shot.

**c.** What is the probability Jeremy will make his next three shots? 1 mark

2 0.3 0.6 0.108 × =

**d.** What is the probability Jeremy will make exactly two of his next three shots? 3 marks

Pr(make two shots) Pr( ') Pr( ' ) Pr( ' ) Pr(make two shots) 0.3 0.6 0.4 0.3 0.4 0.3 0.7 0.3 0.6 Pr(make two shots) 0.234*MMM MM M M MM* = + +

= × × + × × + × × = Two marks for appropriate method, one mark for answer. Note that tree diagram is not essential, but can contribute to appropriate method

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(M2)

(A1)

**e.** Given that Jeremy made at least one of the next three shots, what is the

probability he made all three shots, correct to 2 decimal places? 2 marks Let *X* = number of shots made Pr( *X* = 3 | *X* ≥ 1) = Pr( Pr( *X X* = ≥

1) 3) Pr( *X* = 3 | *X*

≥ 1) = 1 0.108 − 0.7

3

Pr( *X* = 3 | *X*

≥ 1) =

0.16(M1) (A1)

The ATAR scores of applicants for a particular university course are normally distributed. The university decided to accept the top 15% of applicants. Jeremy was accepted into the university with an ATAR score of 86. Only 10% of applicants had a better score than him. His friend Jody was offered a scholarship. Her ATAR score was 95 and only 1% of applicants had a better score than hers.

**f.** What was the mean and standard deviation of the ATAR scores of applicants?

Give your answers correct to two decimal places. 4 marks Pr( *X*< 0.9) =

86 Pr( *Z*< 0.9) =

1.28155 Pr( *X*< 0.99) =

95 Pr( *Z*

< 0.99) =2.32625

1.28155

=

86 σ

− μ

2.32625

=

95 σ

− μ

μ = 74.96, σ

=

8.61

(M1)

(M1)

(A1) + (A1)

Jody wants to join one of the university’s sports clubs. The probability she will be selected for soccer and not rugby is *p*. The probability she will selected for rugby is *p* 3and the probability she will be selected for neither sport is *p* 2. Being selected for soccer is independent of being selected for rugby.

**g.** What is the probability Jody will be selected for both sports? Give your answer

correct to two decimal places. 3 marks

1 *p p p p*

0.54 Pr( *S R* ) *a* Pr( *S* ) Pr( *R*

) *a* ( *a p* )

*p a*

0.10 + + = =

∩ = = × = + =

(M1) 3 2

(M1)

3

(A1) **END OF QUESTION AND ANSWER BOOKLET**

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