SUZANNE CORY
High School

Name : Solutions Teacher : ______________

2016 MATHEMATICAL METHODS (CAS) UNIT 4

SAC 3 – Probability Analysis Task

Writing time: 40 minutes

QUESTION AND ANSWER BOOKLET

Structure of Book

Number of Questions	*Number of questions to be answered*	*Number of marks*
2	2	32

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, and aids for curve sketching.
- Students are NOT permitted to bring into the test room: blank sheets of paper and/or white out liquid/tape any notes or CAS and/or scientific calculator.

Materials supplied

- Question and answer book.

Instructions

- Write your **name** and **teacher** in the space provided above on this page.
- All written responses must be in English.

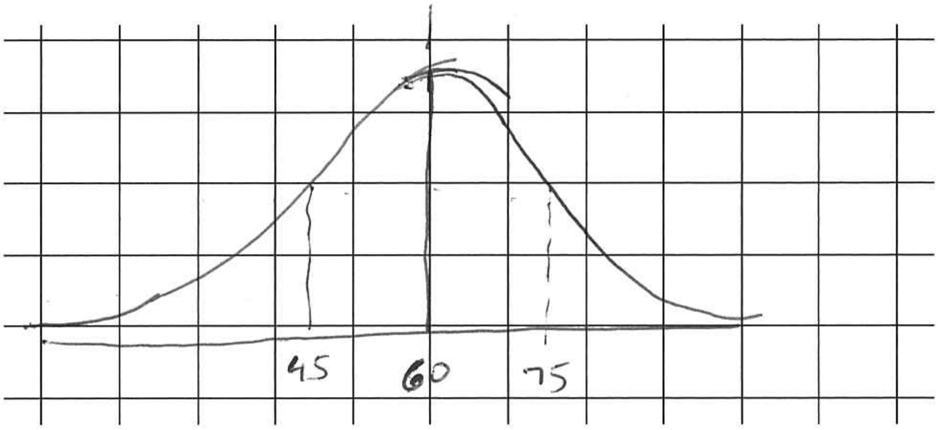
Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.

Question 1 (14 marks)

In 2015, at Suzanne Cory High School, mid-year results in Math Methods were normally distributed with a mean of 60 and standard deviation of 15.

a. Sketch a graph of this distribution labeling the mean and one standard deviation from the mean ($\mu \pm \sigma$).

2 marks



1 mark for Shape

1 mark for Correct μ and σ

b. If a letter grade of a C was given for a score between 60 and 75, determine the **approximate** percentage of students awarded a C to the nearest whole percent?

1 mark

~~32%~~ 34%

c. A support program was put into place for students who needed help with fundamentals of maths methods and also to challenge students for whom the work was too simple. It was decided that to be eligible for this program students needed to score either less than 45 or more than 90. What **approximate** percentage of students would be eligible for the program to the nearest whole percent?

2 marks

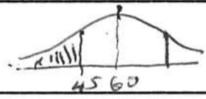
$Pr(x < 45) + Pr(x > 90)$

$16\% + 2.5\%$ } 1m

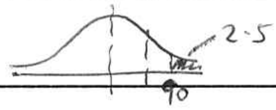
$= 18.5\%$

$\approx 19\%$ — 1m

$Pr(x < 45)$ ~~= 1 − 50%~~



$50\% - 34\% = 16\%$



d. At the end of the year students are eligible to receive academic awards. These are based on mid-year - results and end-of-year results. End of year results were results normally distributed with the vast majority of students scoring 63 and standard deviation of 12.2.

i. What conclusions can be made about the end of year data when compared to the mid-year data?

2 marks

$X_1 \sim N(60, 15^2) \rightarrow X_2 \sim N(63, \cancel{12.8}\ 12.2^2)$.

1m { Improved mean : 60 to 63 ; Less spread : σ 15 to 12.2

1m ∴ Better results with more consistency

ii. Compare the percentages of students scoring less than 40 for mid-year and for end- of year exam? Show all calculations, giving answers to the nearest percent. 2 marks

$X_1 \sim N(60, 15^2)$ $X_2 \sim N(63, 12.2^2)$

$Pr(X < 40)$ $Pr(X < 40)$

$= 0.0912$ ~~0.2970~~ 0.02970.

$\approx 9\%$ $\approx 3\%$. 1m

∴ The amount of students scoring below 40 decreased by 6% 1m or similar

e. Academic awards are awarded according to the table below. The end-of-year results are assumed to be independent of the mid-year results.

Award	Satisfactory	Credit	Distinction	High Distinction	Encouragement award
Mid-year result	55 to < 60	60 to < 70	70 to < 80	80 and over	30 to 40
End of year result	50 to < 60	60 to < 70	70 to < 85	85 and over	70 and over

Use the above information to determine to 4 decimal places:

i. The probability a student receives a credit for both mid-year and end-of-year exams 2 marks

$Pr(\text{Credit})$ and $Pr(\text{credit})$

$= Pr(60 < x < 70) \times Pr(60 < x < 70)$ 1m

$= 0.247508 \times 0.314059$

$= 0.0777$. 1m

ii. The probability a student receives a High distinction for both mid-year and end-of-year exams 2 marks

$Pr(HD)$ & $Pr(HD)$

$= Pr(80 < x < 100) \times$ ~~Pr(85 < x < Pr...~~ $Pr(85 < x < 100)$ 1M

$= 0.087381 \times 0.034461$

$= 0.0030$. 1M

iii. If there are 167 students, how many will receive an Encouragement Award for both mid-year and end-of-year exams? Answer to the nearest number of students 3 marks

mid-year: $Pr(30 < x_1 < 40) \times 167 = 11.4$

≈ 11 students 1m

$Pr(70 < x_2 < 100) \times 167 = 47.1$

$\approx 47.$ students 1m

Can we reduce to 2 marks?

Question 2 (15 marks)

In 2014, results were also normally distributed, with approximately 68% of students symmetrically scoring between 45.2 and 60.8

a. Show that the 2014 results can be defined by $X \sim N\,(53, 7.8^2)$ 2 marks

68% of results b/w 45.2 & 60.8

Since 68% of results ~~B~~ ~~ff~~ $\mu \pm \sigma$ 1m

$\mu = \frac{45.2 + 60.8}{2}$ $\quad$ $\sigma = 53 - 45.2$

$= 53$ $\quad$ 1m $\quad$ $= 7.8$ Hence $X \sim N(53, 7.8^2)$

b. i If in 2014 a students scored 65, write this as a z score to 2 decimal places. 2 marks

$z = \frac{x - \mu}{\sigma}$ 1m for either

$z = \frac{65 - 53}{7.8} \Rightarrow z = 1.5385$

≈ 1.54 1m

ii In which percentile would this student's score lie? Give your answer to the nearest % 2 marks

$Pr(z < 1.53846) = 0.9380$ 1m

≈ 94th percentile 1m

It is known that another student in 2014 scraped into the top 79%, but their exact results have been lost.

c. Determine their z score and their actual result to 2 decimal places. 2 marks

$Pr(z < a) = 0.21$ 1m

$\Rightarrow a = -0.8064$.

$x = \sigma z + \mu$

$~~-7.8\times0~~ = 7.8 \times -0.8064 + 53$

$= 46.7099$

$= 46.71$ 1m

d. It is expected that in 2016, results will once again be normally distributed with an expected result of 56 and standard deviation of 6. If a student performed in the top 10% in 2014, how would they perform in 2016?

3 marks

$X \sim N(56, 6^2)$ – 2016

For 2014:

$Pr(Z > a) = 0.9$

$\therefore 1 - Pr(Z < a) = 0.1$

$\Rightarrow a = 1.28$ $\boxed{1m}$

$x = z\sigma + \mu$

$= 1.28 \times 7.8 + 53$

$= 62.99$.

62.99 in 2014 → 2016

$z = \frac{62.99 - 56}{6}$

$= 1.16$ $\boxed{1m}$

$Pr(Z > 1.16) = 0.1218$

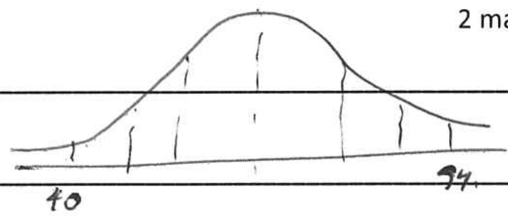
$\Rightarrow$ top 12% $\boxed{1m}$

e. In planning for the future, staff want to see an improvement in students' results, with no students scoring below a mark of 40. **Assume that the proportion of students scoring between 40 and 94 lies within 3 standard deviations of the mean ($\mu \pm 3\sigma$).** In 2017, there are 180 students who will undertake Maths Methods. Staff must be able to work out the expected value and standard deviation as well as the highest mark a student in the bottom 5% score can score and the lowest mark a student in the top 5% can score.

i. Determine the mean and standard deviation 2 marks

$\mu = \frac{40 + 94}{2}$

$= 67$. 1m



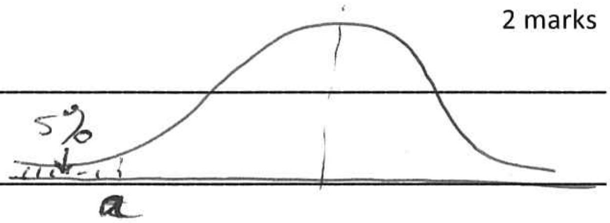
6σ b/w 40 & 94

$\therefore \sigma = \frac{40+94}{6}$

$= 22\frac{1}{3}$ 1m

ii. Calculate the highest possible mark a student in the bottom 5% may achieve (To the nearest whole mark)

2 marks



$z \to x$.

$Pr(z < a) = 0.05$

$\Rightarrow a = -1.6449$. 1 m

$X = \sigma z + \mu$

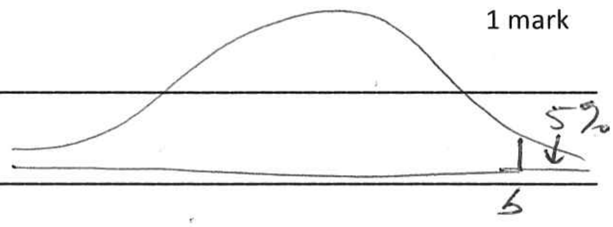
$= 22\frac{1}{3} \times -1.6449 + 67$

$= 30.26$

≈ 30. 1 m

iii. Calculate the lowest possible mark a student in the top 5% may achieve. (To the nearest whole mark)

1 mark



Due to symmetry:

$b = 1.6449$

$\therefore X = \sigma z + \mu$

$= 22\frac{1}{3} \times 1.6449 + 67$

$= 99.74$

≈ 100 1m.

END OF QUESTION AND ANSWER BOOKLET