Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.

Question 1 (14 marks)

At Suzanne Cory High School students have the opportunity to participate in a number of activities as part of their school life. Let X be a discrete random variable representing the number of different activities that a junior class (year 9 and year 10) takes part in. The table below shows the probability distribution for X:

X	2	3	4	5	6
$\Pr(X = x)$	$\frac{2}{3+k}$	$\frac{k}{2}-1$	$\frac{k^2}{30}$	$\frac{1}{8}$	$\frac{1}{16}$

Where $k \in R^+$

a. Find the value of k correct to two decimal places. 2 marks

$$\frac{2}{3+k}+\frac{k}{2}-1+\frac{k^2}{30}+\frac{1}{8}+\frac{1}{16}=1$$ (M1)

$k = 2.48$ (A1)

Let Y be a discrete random variable representing the number of different activities that any senior class (year 11 and year 12) takes part in. The table below shows the probability distribution for Y:

Y	2	3	4	5	6
$\Pr(Y = y)$	0.19	0.27	0.35	0.12	0.07

b. Find the probability, correct to two decimal places, that a senior class:

i. Participates in more than 3 activities. 1 mark

$\Pr(Y > 3) = 0.35 + 0.12 + 0.07$

$= 0.54$ (A1)

ii. Participates in at least 5 activities given that they participate in more than 3 activities. 2 marks

$$\Pr(Y \geq 5 \backslash Y > 3) = \frac{\Pr(Y \geq 5 \cap Y > 3)}{\Pr(Y > 3)}$$

$$= \frac{\Pr(Y \geq 5)}{\Pr(Y > 3)} \text{ or } \frac{\Pr(Y \geq 5)}{\Pr(Y \geq 4)}$$

$= \frac{0.19}{0.54}$ (M1)

$= 0.35$ (A1)

c. Find (correct to two decimal places)

i. the mean
ii. the median
iii. the mode
iv. the variance 4 marks

$E(Y) = 2(0.19) + 3(0.27) + 4(0.35) + 5(0.12) + 6(0.07)$

$= 3.61$ (A1)

$median = 4$ (A1)
$Mode = 4$ (A1)
$VAR(Y) = 1.28$ (A1)

d. Find $\Pr(\mu - \sigma \le Y \le \mu + 3\sigma)$ correct to two decimal places. 2 marks

$\Pr(\mu - \sigma \le Y \le \mu + 3\sigma) = \Pr(2.4796 \le Y \le 7.0013)$

$= \Pr(3 \le Y \le 7)$ (M1)

$= 0.81$ (A1)

e. A senior class is randomly selected out of 16 classes. Find the probability, correct to two decimal places that 12K2 participates in at most 3 activities. 2 marks

$\Pr(12K2 \cap Y \le 3) = \frac{1}{16} \times 0.46$ (M1)

$= 0.03$ (A1)

f. Find the probability that out of 16 senior classes more than half participate in at least three activities. 1 mark

$\Pr(X > 8) = 0.9949$

Note: Binomial Cdf

Question 2 (8)

There are two types of activities (physical and mental). Let A denote the event that a boy is selected and B denote the event that a student enjoys physical activities. It is known that boys occupy 60% of the whole cohort and the probability that a boy likes physical activity is 30%. Let **x** be the probability that a randomly selected student prefers mental activities.

a. Represent the information using a Karnaugh map. 2 marks

	A	A'	
B	0.3	0.7 – x	1 - x
B'	0.3	x – 0.3	x
	0.6	0.4	1

All correct: 2 marks
5 or more correct: 1 mark
Less than 5 correct: 0 mark

b. It is known that the probability that a randomly selected student prefers physical activity is equal to the square of that who likes mental activity. Find the value of x (correct to two decimal places).

2 marks

$1-x=x^2$ (M1)

$x=0.62$ (A1)

c. Find the probability that a randomly selected student who likes physical activities is a girl (correct to two decimal places). 2 marks

$\Pr(A'\backslash B)=\dfrac{\Pr(A'\cap B)}{\Pr(B)}$

$=\dfrac{0.7-x}{1-x}$ (M1)

$=\dfrac{0.7-0.62}{1-0.62}$

$=0.21$ (A1)

d. Are A and B independent? Justify your answer. 2 marks

$\Pr(A\cap B)=0.3$

$\Pr(A)\times\Pr(B)=0.6\times(1-0.6180)$

$=0.2292$

A and B are not independent (A1)

Because $\Pr(A\cap B)\neq\Pr(A)\times\Pr(B)$ (A1)

Note: Students must show the value of $\Pr(A)\times\Pr(B)$

Question 3 (8)

There are 167 students enrolled in Mathematical Methods 3&4 at Suzanne Cory in 2016. The probability that a randomly selected student loves the subject is 0.3. Let X be the number of students who love Maths Methods

a. What is the expected number of students who love Maths Methods? 1 mark

$E(X)=np$

$=167\times0.3$

$=50.1$ (A1)

b. Find the standard deviation of X correct to two decimal places. 1 mark

$SD(X)=\sqrt{np(1-p)}$

$=\sqrt{167\times0.3\times0.7}$

$=5.92$ (A1)

c. Find the probability that more than 25% of students love Maths methods correct to two decimal places. 1 mark

$\Pr(X\geq 42)=0.93$ (A1)

d. 10 Students are randomly selected. Find the probability that only the first and the last students love Maths Methods correct to four decimal places. 2 marks

$\Pr(First\ and Last) = 0.3 \times 0.7^8 \times 0.3$ (M1)

$= 0.0052$ (A1)

e. Every year the number of enrolments changes slightly. However, we can assume that the probability that a student obtains a perfect ATAR score is 15%. What is the least number of enrolments required to make sure that the probability that at least two students will have this ATAR is more than 90%? 3 marks

$\Pr(X \geq 2) \geq 0.9$

$1 - \Pr(X = 0) - \Pr(X = 1) \geq 0.9$ (M1)

$1 - 0.85^n - \binom{n}{1}(0.15)(0.85)^{n-1} \geq 0.9$

$1 - 0.85^n - n(0.15)(0.85)^{n-1} \geq 0.9$ (M1)

$n \geq 25$

The least number is 25 (A1)

Question 4 (14)

The amount of time each student spends on homework varies with year levels. The average amount of time X (in hours) each student spends on homework every night has a probability density function defined by

$$f(x) = \begin{cases} g(x) = a\left[2\sin(x) + 4\right], 0 \leq x < \pi \\ h(x) = \dfrac{a}{\pi}\left[-2x + 6\pi\right], \pi \leq x \leq 2\pi \\ 0, \text{elsewhere} \end{cases}$$

where a is a constant

a. Use calculus to show that $a = \dfrac{1}{7\pi + 4}$. **Do not use CAS** 3 marks

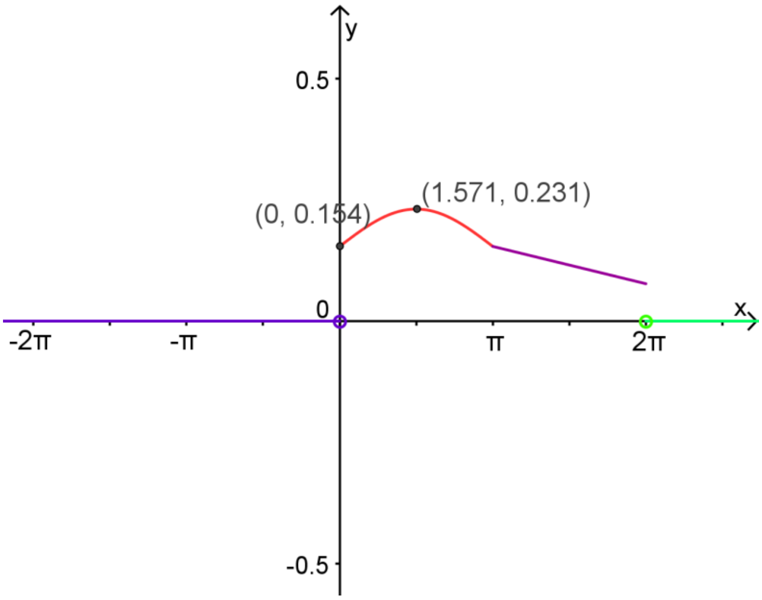
$\int_0^{\pi} a[2\sin(x) + 4]\,dx + \int_{\pi}^{2\pi} \frac{a}{\pi}\left[-2x + 6\pi\right]dx = 1$ (M1)

$a\left[-2\cos(x) + 4x\right]_0^{\pi} + \frac{a}{\pi}\left[-x^2 + 6\pi x\right]_{\pi}^{2\pi} = 1$ (M1)

$a\left[2 + 4\pi + 2 - 0\right] + \frac{a}{\pi}\left[-4\pi^2 + 12\pi^2 + \pi^2 - 6\pi^2\right] = 1$

$a \times [7\pi + 4] = 1$ so $a = \dfrac{1}{7\pi + 4}$ as required (A1)

b. Sketch the graph of f(x) versus x labelling any intercepts and turning point using coordinates. Give all values correct to three decimal places. 2 marks



(0, 0.154)	M ½
(1.571, 0.231)	M ½
Shape	M ½
Two horizontal lines with opened circle at one end	M ½

c. Find

i. The mean time (to the nearest minute) each student spends on homework. 2 marks

$$\mu = \int_0^{\pi} x \times g(x)\,dx + \int_{\pi}^{2\pi} x \times h(x)dx \qquad \text{(M1)}$$

$$= 2\,\text{hours}\,39\,\text{minutes} \qquad \text{(A1)}$$

ii. The interquartile range (to the nearest minute). 2 marks

$$\int_0^{m} f(x)\,dx = 0.25$$

m = 1 hour 16 minutes

$$\int_0^{\pi} g(x)\,dx + \int_{\pi}^{n} h(x)\,dx = 0.75$$

n = 3 hours 55 minutes (M1) both m and n

IQR = 2 hours 39 minutes (A1)

iii. The mode (to the nearest minute) 1 mark

mode = 1 hour 34 minutes (A1)

iv. The standard deviation (to the nearest minute) 2 marks

$$\sigma^2 = \int_0^{\pi} (x-\mu)^2 \times g(x)\,dx + \int_{\pi}^{2\pi} (x-\mu)^2 \times h(x)\,dx \text{ or } \sigma^2 = E(X^2) - \mu^2$$

$= 2.7915.....$ (M1)

$\sigma = 1.6708$ hours

$= 1\,\text{hour}\,40\,\text{minutes}$ (A1)

v. If a student spends at most 4 hours on homework, find the probability that he/she spends at least 2 hours on homework. Give your answer correct to two decimal places 2 marks

$$\Pr(X \geq 2 \setminus X \leq 4) = \frac{\Pr(X \geq 2 \cap X \leq 4)}{\Pr(X \leq 4)}$$ (M1)

$$= \frac{\Pr(2 \leq X \leq 4)}{\Pr(X \leq 4)}$$

$$= \frac{\int_2^{\pi} g(x)\,dx + \int_{\pi}^{4} h(x)\,dx}{\int_0^{\pi} g(x)\,dx + \int_{\pi}^{4} h(x)\,dx}$$

$= 0.45$ (M1)