SUZANNE CORY
High School

Name : ____________________ Teacher : __________________

2016 MATHEMATICAL METHODS (CAS) UNIT 3

SAC 3 – Application Task: Part 2 of 2

Chapters 10, 11, 12, 13: Probability

Reading time: 10 minutes
Writing time: 70 minutes

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Section	*Number of Questions*	*Number of questions to be answered*	*Number of Marks*
A	*4*	*4*	*44*

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference (which will be collected for the duration of the SAC), one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the test room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Information sheet
- Formula sheet
- Question and answer booklets

Instructions

- Write your **name** and **teacher** in the space provided above on both the information sheet and the booklet.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.

Instructions
Answer all questions in the spaces provided. In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. In questions where more than one mark is available, appropriate working **must** be shown.

Question 1 (14 marks)

At Suzanne Cory High School students have the opportunity to participate in a number of activities as part of their school life. Let X be a discrete random variable representing the number of different activities that a junior class (year 9 and year 10) takes part in. The table below shows the probability distribution for X:

X	2	3	4	5	6
$\Pr(X = x)$	$\frac{2}{3+k}$	$\frac{k}{2}-1$	$\frac{k^2}{30}$	$\frac{1}{8}$	$\frac{1}{16}$

Where $k \in R^+$

a. Find the value of k correct to two decimal places. 2 marks

Let Y be a discrete random variable representing the number of different activities that any senior class (year 11 and year 12) takes part in. The table below shows the probability distribution for Y:

Y	2	3	4	5	6
$\Pr(Y = y)$	0.19	0.27	0.35	0.12	0.07

b. Find the probability, correct to two decimal places, that a senior class:

i. Participates in more than 3 activities. 1 mark

ii. Participates in at least 5 activities given that they participate in more than 3 activities.
2 marks

c. For the variable Y, find (correct to two decimal places):

i. the mean 1 mark

ii. the median 1 mark

iii. the mode 1 mark

iv. the variance 1 mark

d. Find $\Pr(\mu - \sigma \le Y \le \mu + 3\sigma)$ correct to two decimal places. 2 marks

e. A senior class is randomly selected out of 16 classes. Find the probability, correct to two decimal places that 12K2 participates in at most 3 activities. 2marks

f. Find the probability that out of 16 senior classes more than half participate in at least three activities.
1 mark

Question 2 (8)

There are two types of activities (physical and mental). Let A denote the event that a boy is selected and B denote the event that a student enjoys physical activities. It is known that boys occupy 60% of the whole cohort and the probability that a boy likes physical activity is 30%. Let **x** be the probability that a randomly selected student prefers mental activities.

a. Represent the information using a Karnaugh map. 2 marks

b. It is known that the probability that a randomly selected student prefers physical activity is equal to the square of that who likes mental activity. Find the value of x (correct to two decimal places).

2 marks

c. Find the probability that a randomly selected student who likes physical activities is a girl (correct to two decimal places). 2 marks

d. Are A and B independent? Justify your answer. 2 marks

Question 3 (8)

There are 167 students enrolled in Mathematical Methods 3&4 at Suzanne Cory in 2016. The probability that a randomly selected student loves the subject is 0.3. Let X be the number of students who love Maths Methods

a. What is the expected number of students who love Maths Methods? 1 mark

b. Find the standard deviation for X, correct to two decimal places. 1 mark

c. Find the probability that more than 25% of students love Maths methods correct to two decimal places. 1 mark

d. 10 Students are randomly selected. Find the probability that only the first and the last students love Maths Methods correct to four decimal places. 2 marks

e. Every year the number of enrolments changes slightly. However, we can assume that the probability that a student obtains a perfect ATAR score is 15%. What is the least number of enrolments required to make sure that the probability that at least two students will have this perfect ATAR is more than 90%? 3 marks

Question 4 (14)

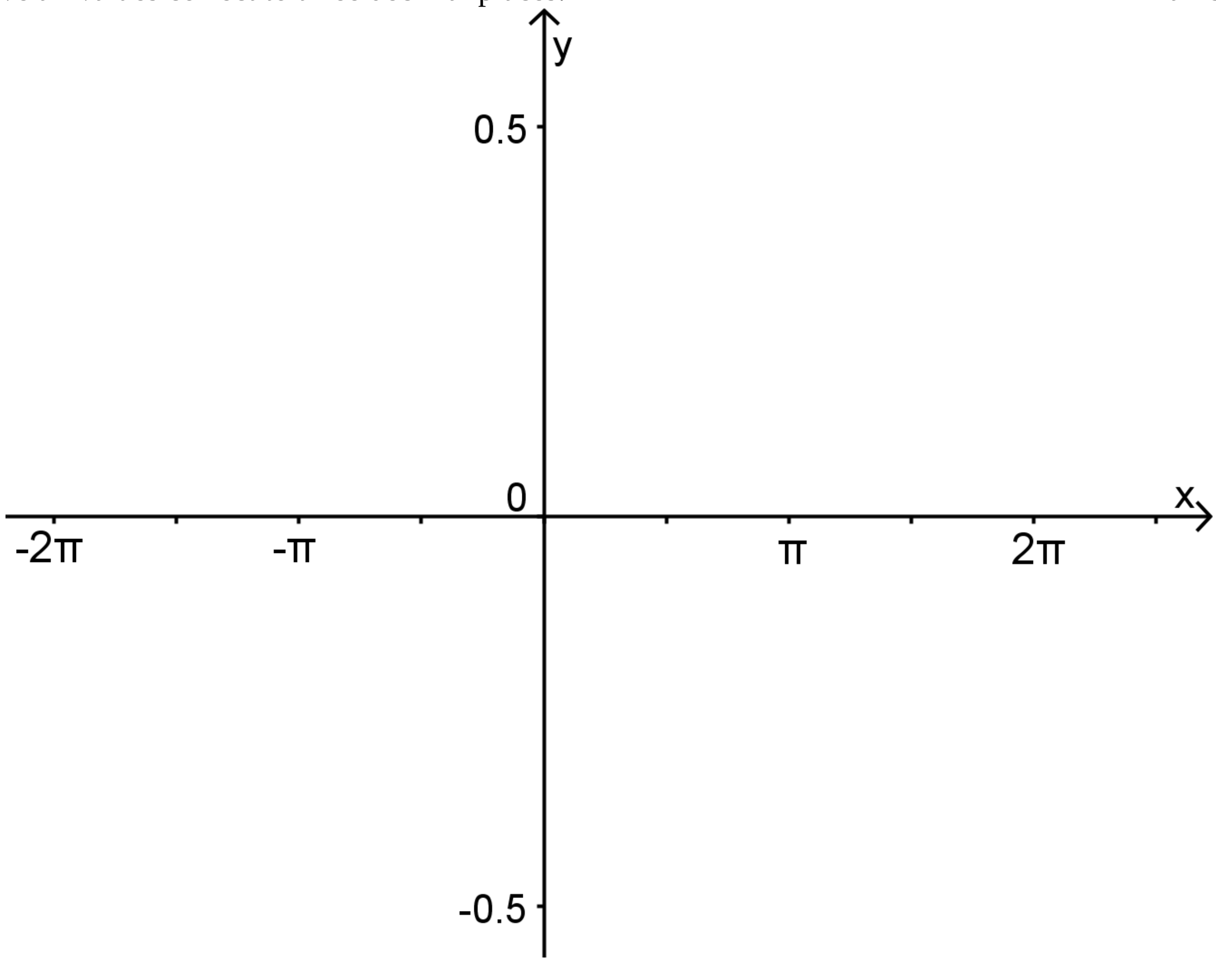
The amount of time each student spends on homework varies with year levels. The average amount of time X (in hours) each student spends on homework every night has a probability density function defined by

$$f(x)=\begin{cases} g(x)=a\left[2\sin(x)+4\right], 0\le x<\pi \\ h(x)=\dfrac{a}{\pi}\left[-2x+6\pi\right], \pi\le x\le 2\pi \\ \quad 0, \text{elsewhere} \end{cases}$$

where a is a constant

a. Use calculus to Show that $a=\dfrac{1}{7\pi+4}$. **Do not use CAS** 3 marks

b. Sketch the graph of f(x) versus x labelling any intercepts and turning point using coordinates. Give all values correct to three decimal places. 2 marks



c. Find

i. The mean time (to the nearest minute) each student spends on homework. 2 marks

ii. The interquartile range (to the nearest minute). 2 marks

iii. The mode (to the nearest minute) 1 mark

iv. The standard deviation (to the nearest minute) 2 marks

v. If a student spends at most 4 hours on homework, find the probability that he/she spends at least 2 hours on homework. Give your answer correct to two decimal places. 2 marks

END OF QUESTION AND ANSWER BOOKLET

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\vert B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\mathrm{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \frac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$