



Name : _____ Teacher : _____

2017 MATHEMATICAL METHODS (CAS) UNIT 4

SAC 3 – Probability Analysis Task

NON-TEST CONDITIONS PART 1

Writing time: 40 minutes

QUESTION AND ANSWER BOOKLET

Structure of Book

<i>Number of Questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
2	2	23

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, one approved CAS calculator, one scientific calculator, one bound reference, rulers, a protractor, set-squares, aids for curve sketching.

Materials supplied

- Question and answer book.

Instructions

- Write your **name** and **teacher** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.

Question 1 (10 marks)

- a. Find the probability that 5 funnels will be defective. Give your answer to four decimal places. (1 mark)

A1 Correct answer
0.0208

- b. Find the probability that at least 10 funnels (10 or more) will not be defective. Give your answer correct to four decimal places. (1 mark)

A1 Correct answer
0.9943

- c. Show that the expected number of defective funnels is 1.8 (1 mark)

M1 Correct application of binomial expectation
 $5 \times 0.12 = 1.8$

- d. Find the mean cost of operating the assembly line and manufacturing the non-defective funnels. Give your answer to the nearest whole number. (2 marks)

M1 Correctly set-up the relationship between Y and T , or correctly set-up the relationship between $E(Y)$ and $E(X)$. Also allow $Y=1000+150Z$, $Z=\#$ non-defective
 $Y = 3250 - 150X$ or $E(Y) = 3250 - 150E(X)$

A1 Correct answer
\$2980

All of the funnels produced are sent to the warehouse, where they await to be shipped to clients. One of the managers of Clover Pipelines wishes to estimate how many defective funnels are in the warehouse. It is assumed that 12% of all funnels in the warehouse will be defective. A sample of 8 funnels are randomly selected. Let \hat{P} be the random variable of the distribution of sample proportions of defective funnels.

- e. Find $\Pr(0.25 < \hat{P} < 0.75)$. Give your answer to 4 decimal places. Do not use the normal approximation. (2 marks)

M1 Convert probability involving \hat{P} into a probability X , or an appropriate alternative strategy
 $\Pr(0.25 < \hat{P} < 0.75) = \Pr(2 < X < 6)$, or equivalent

A1 Correct answer
0.0607

- f. i. Find the standard deviation $\sigma_{\hat{p}}$. Give your answer to 4 decimal places. (1 mark)

A1 Correct answer
0.0103

- ii. Using your result from part g. (i), use a normal approximation to find $\Pr(\hat{P} > 1/8)$. Give your answer to 4 decimal places. (1 mark)

A1 Correct answer
0.3137

- g. In a random sample of 500 funnels, 65 were found to be defective. Determine a 95% confidence interval for p . Give your answer to 4 decimal places. (1 mark)

A1 Correct answer
(0.1005, 0.1595)

Question 2 (16 marks)

The masses of the funnels can be described by the following probability density function

$$f(x) = \begin{cases} kx(x-4)(x-5), & 0 \leq x \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

- a) Show that the value of k is $\frac{1}{32}$ (1 mark)

M1 Demonstrate that the sum of the probabilities equals 1.
If the student show additional working it must be correct, otherwise the student obtains 0.

Accept: $\int_0^4 f(x) dx = 1$. Solving for k gives $k = \frac{1}{32}$

- b) Calculate the mean weight (expected value). Give you answer as an exact value. (2 marks)

M1 Correctly apply formula for calculating the mean
 $\int_0^4 xf(x) dx$

A1 Correct answer
 $\frac{26}{15}$

Let m be the median of the distribution.

- c) i. Find the values of a and b which satisfy the equation $m^4 - am^3 + bm^2 - 64 = 0$. (2 marks)

M1 Correctly set up equation to find the median
 $\int_0^m f(x) dx = \frac{1}{2}$, or equivalent

A1 Correct answer
 $a = 12$ and $b = 40$

ii. Hence, find the value of m . Give your answer to 2 decimal places. (1 mark)

A1 Correct answer
1.68

d) A funnel with a mass below 1kg is classified as 'light'. Find the proportion of funnels that are classified as 'light'. Give your answer to four decimal places. (1 mark)

A1 Correct answer
0.2266

e) The heaviest 20% of funnels are classified as 'heavy'. Find the smallest possible mass of a 'heavy' funnel. Give your answer correct to four decimal places. (2 marks)

M1 Construct a correct equation that allows a student to find the unknown value.
Example: $\int_k^4 f(x)dx = 0.2$, or equivalent

A1 Correct answer
 $k = 2.5225$

f) Given that a funnel weights more than 1.6 kg, find the probability that a funnel weights less than 2.8 kg. Give your answer correct to four decimal places. (2 marks)

M1 Construct a correct conditional probability expression, or equivalent
Example: $\Pr(X < 2.8 | X > 1.6)$
Do not penalise students for using $>$ or \geq etc.

A1 Correct answer
0.7601

g) Find the minimum value of p that satisfies this condition. Give your answer correct to 3 decimal places. (2 marks)

M1 Demonstrate some recognition that the binomial distribution must be used
Example: $Y \sim Bi(10, p)$ or $\Pr(Y = 10) \geq 0.8$

A1 Correct answer
 $p \geq 0.978$ or $p = 0.978$