



2017 MATHEMATICAL METHODS (CAS) UNIT 4

**SAC 3 – Probability Analysis Task**

**NON-TEST CONDITIONS PART 1**

**Writing time: 40 minutes**

### QUESTION AND ANSWER BOOKLET

### Structure of Book

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| --- | --- | --- |
| *Number of*  *Questions* | *Number of questions*  *to be answered* | *Number of*  *marks* |
| 2 | 2 | 23 |

⦁ Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, one approved CAS calculator, one scientific calculator, one bound reference, rulers, a protractor, set-squares, aids for curve sketching.

**Materials supplied**

⦁ Question and answer book.

**Instructions**

⦁ Write your **name** and **teacher** in the space provided above on this page.

⦁ All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.**

**Question 1 (10 marks)**

Clover Pipelines is an industrial funnel manufacturer located in Cambellfield, in Victoria’s northern suburbs. They make a wide range of funnels. Due to their irregular shape, only 15 funnels are made each day. There is a 12% chance that a funnel that is produced will be defective (faulty). Each funnel produced is independent of the previous funnel produced. Let represent the number of defective funnels that are manufactured.

1. Find the probability that 5 funnels will be defective. Give your answer to four decimal places. (1 mark)

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1. Find the probability that at least 10 funnels (10 or more) will not be defective. Give your answer correct to four decimal places. (1 mark)

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1. Show that the expected number of defective funnels is 1.8 (1 mark)

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To manufacture funnels, Clover Pipelines must pay a cost to operate the assembly line and incur a cost for each non-defective funnel produced. Let the represent the cost of operating the assembly line and manufacturing the non-defective funnels.

It costs $1000 to run the assembly line each day, plus $150 for each non-defective funnel.

1. Find the mean cost of operating the assembly line and manufacturing the non-defective funnels. Give your answer to the nearest whole number. (2 marks)

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All of the funnels produced are sent to the warehouse, where they await to be shipped to clients. One of the managers of Clover Pipelines wishes to estimate how many defective funnels are in the warehouse. It is assumed that 12% of all funnels in the warehouse will be defective. Samples of 8 funnels are randomly selected. Let be the random variable of the distribution of sample proportions of defective funnels.

1. Find . Give your answer to 4 decimal places. Do not use the normal approximation. (2 marks)

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The manager is determined to get more accurate information and assigns several employees to go and select funnels from the warehouse. The sample size has now increased from 8 to 1000.

1. i. Find the standard deviation . Give your answer to 4 decimal places. (1 mark)

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ii. Using your result from part f. (i), use a normal approximation to find . Give your answer to 4 decimal places. (1 mark)

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1. In a random sample of 500 funnels, 65 were found to be defective. Determine a 95% confidence interval for . Give your answer to 4 decimal places. (1 mark)  
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**Question 2 (13 marks)**

The masses of the funnels can be described by the following probability density function

where is a constant.

1. Show that the value of is (1 mark)

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1. Calculate the mean weight (expected value). Give you answer as an exact value. (2 marks)

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Let be the median of the distribution.

1. i. Find the values of and which satisfy the equation .

(2 marks)

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ii. Hence, find the value of . Give your answer to 2 decimal places. (1 mark)

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1. A funnel with a mass below 1kg is classified as ‘light’. Find the proportion of funnels that are classified as ‘light’. Give your answer to four decimal places. (1 mark)

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1. The heaviest 20% of funnels are classified as ‘heavy’. Find the smallest possible mass of a ‘heavy’ funnel. Give your answer correct to four decimal places. (2 marks)

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1. Given that a funnel weighs more than 1.6 kg, find the probability that a funnel weighs less than 2.8 kg. Give your answer correct to four decimal places. (2 marks)

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Clients of the manufacturer now require a different funnel, *the soul catcher*, that Clover Pipelines does not make. Clover Pipelines decide to import *the soul catcher.* They are sold in groups of 10. The manager wants to be at least 80% certain that in a group of 10 *soul catcher* funnels all will have a mass over 1 kg. Let be the proportion of funnels over 1 kg.

1. Find the minimum value of that satisfies this condition. Give your answer correct to 3 decimal places. (2 marks)

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