



Name: _____ Teacher: _____

2018 MATHEMATICAL METHODS UNIT 4

SAC 3: Part 1

Chapters 10, 11, 12, 13 – Probability

Reading time: 0 minutes
Writing time: 40 minutes

QUESTION AND ANSWER BOOKLET

Structure of Booklet

<i>Number of Questions</i>	<i>Number of questions to be answered</i>	<i>Number of Marks</i>
2	2	17

I suggest 19 or 20 marks to prepare students for the 2 marks Qs in the CAT

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator and, if desired, one scientific (non-programmable) calculator. For approved computer-based CAS, its full functionality may be used and its memory needs not be cleared at the start of the assessment.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape nor make any notes on their bound reference.
- Failure to comply with any of the above will be regarded as cheating and the offenders will be sanctioned according to VCAA rules.

Materials supplied

- Question and Answer booklet.

Instructions

- Write **your full name** and your **teacher's name** in the space provided above.
- Unless otherwise stated, all numerical answers must be given as **exact values**.
- Answer all questions in the spaces provided.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Mr Chau expected the team to get through the group stages and play in the division final. Alex plays a total of 4 games.

d. What is the probability that Alex will score exactly two goals in a game in two of the four games played?

$Y = \text{No. of games Alex scores 2 goals}$ (1 mark)

$$P(X=2) = 0.15$$

$$Y \sim \text{Bi}(4, 0.15)$$

$$P(Y=2) = 0.097538$$

$$\frac{7803}{80000}$$

e. What is the probability that Alex will score two goals in a game in at least three of the four games?

1 mark

$$P(Y \geq 3) = 0.011981$$

$$\frac{1917}{160000}$$

f. What is the probability that Alex will score two goals in a game in at least three of the four games, given that he scored two goals in the first game?

(1 mark)
2 marks unless you want an answer only

$$P(Y \geq 3 | Y = 2^{\text{th}} \text{ game}) = \frac{\frac{1917}{160000} - 0.85 \times 0.15^3}{0.15}$$

$$= 0.06075$$

$$\frac{243}{4000}$$

Question 2 (9 marks):

Games at the Wyndham division finals were 25mins long. In a further analysis, Mr Chau constructed a probability density function for the number of minutes, Y , which Alex will play for each game. The probability density function can be described by the function:

$$f(t) = \begin{cases} -k \cos\left(\frac{\pi t}{15}\right) + k, & 0 \leq t \leq 25 \\ 0, & \text{otherwise} \end{cases}$$

Where t is in minutes, and k is a positive real number.

a. Show that k is approximately 0.0343, when rounded to four decimal places.

1 mark

$$\int_0^{25} f(t) dt = 1$$

Use this approximate value of k for the remaining questions.

could you move it to next page above part b.

Alex is SCHS's star striker of their soccer team. He was expected to carry the team through the Wyndham Division to qualify for the regionals! Their coach, Mr Chau, wanted to do a probability analysis of Alex's goal scoring prowess.

Question 1 (8 marks):

Based on previous data, the discrete random variable number X for the number of goals that Alex scores per game is given by the probability distribution table below:

x	0	1	2
$\Pr(X = x)$	0.3	a	0.15

a. Show a is 0.55.

1 mark

$$0.3 + a + 0.15 = 1$$

$$\therefore a = 0.55$$

b. Calculate expected number of goals Alex will score in a game

2 marks

$$E(X) = 0 \times 0.3 + 1 \times 0.55 + 2 \times 0.15$$

$$= 0.55 + 0.3$$

$$= 0.85 \text{ goals}$$

$$\frac{17}{20}$$

c. Calculate $\text{var}(X)$, the variance of X .

(d.p)

2 marks

$$\text{var}(X) = E(X^2) - [E(X)]^2$$

$$= (0^2 \times 0.3 + 1^2 \times 0.55 + 2^2 \times 0.15) - 0.85^2$$

$$= 0.4275$$

$$\frac{171}{400}$$

b. Find the probability that Alex will play at least 20 minutes in a game? ⁴ d.p. 1 mark

$$= 5k$$

$$= 0.1715$$

$$\text{or } 0.1716$$

if k is not rounded

c. Find the median number of minutes Alex plays per game. $k = 0$ 2 marks

$$\int_0^m f(t) dt = 0.5$$

when $k = 0.0343$

$$m = 14.7886$$

if k is not rounded

$$m = 14.7837$$

Alex's teammate Karthik disagrees with the model. He does not think Alex spends that much time on the pitch in interschool competition. He thinks that Alex's game time is better modelled by a random **normally distributed** variable W , where W is the number of minutes Alex plays per game. Karthik's proposed normal distribution has a mean of 12.5 minutes but an unknown standard deviation.

Given that the probability that Alex plays at most 20 minutes in a game is 0.8.

d. Show that, according to Karthik's model, the standard deviation of Alex's game time, $sd(W)$, is 8.911, when rounded to three decimal places. 2 marks

$$Pr(W \leq 20) = 0.8$$

$$0.841621 = \frac{20 - 12.5}{\sigma}$$

$$Pr(Z \leq z_1) = 0.8$$

$$\sigma = 8.911$$

$$z_1 = 0.841621$$

e. Using the standard deviation ^{the} calculated in (2d), calculate ^{d.p.} probability that Alex plays for at least 16 minutes in a particular game. 1 mark

$$Pr(W \geq 16) = 0.3472$$

Of the 4 games Alex plays

The probability that Alex plays more than m minutes per game is given by p . The probability that Alex will play more than m minutes per game in 2 of the 4 games is given by q .

(In this case, q is not defined as $1 - p$).

f. Write down the polynomial of q in terms of p in expanded form.

2 marks
1 mark

Let M be the number of games Alex plays more than m minutes

$$M \sim \text{Bi}(4, p)$$

$$q = 6p^4 - 12p^3 + 6p^2$$

$$\text{Pr}(M=2) = q$$

$$q = {}^4C_2 p^2 (1-p)^2$$

g. Find the maximum value of q .

1 mark
2 marks

$$\frac{dq}{dp} = 0$$

$$\text{max at } p = \frac{1}{2}$$

$$p=0, p=\frac{1}{2}, p=1$$

$$q = \frac{3}{8}$$

As it turns out, on match day, all that calculation was a waste because the only thing Alex was carrying after the first game was an injury!

END OF SAC 3 PART 1