SUZANNE CORY
High School

Name: SOLUTIONS **Teacher:** CCH

2018 MATHEMATICAL METHODS UNIT 4

SAC 3: Part 2

Chapters 10, 11, 12, 13 – Probability

Reading time: 10 minutes
Writing time: 75 minutes

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Number of Questions	*Number of questions to be answered*	*Number of Marks*
4	4	34

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator and, if desired, one scientific (non-programmable) calculator. For approved computer-based CAS, its full functionality may be used and its memory needs not be cleared at the start of the assessment.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape nor make any notes on their bound reference.
- Failure to comply with any of the above will be regarded as cheating and the offenders will be sanctioned according to VCAA rules.

Materials supplied

- Question and Answer booklet.

Instructions

- Write **your full name** and your **teacher's name** in the space provided above.
- Unless otherwise stated, all numerical answers must be given as **exact values**.
- Answer all questions in the spaces provided.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Hang onto your CAS's when you are in Suzanne Cory High School! There have been reports that CAS's are going missing!

Question 1 (9 marks):
The number of CAS's reported missing have increased in 2018.

The number of CAS's that are reported missing each week is assigned the random variable A and can be described by the discrete probability distribution as shown below:

a	0	1	2
$\Pr(A = a)$	$1.5k$	$2.5k$	$5k^2$

a. Show that the value of k is 0.2. Justify in words, or otherwise, why one of the k values is rejected. 2 marks

$k + 3k + 5k^2 = 1$
$k = 0.2 \; or \; k = -1$ [A1]
k cannot be a negative number, as probabilities in a distribution table cannot be negative. [A1]

b. State the value of $\Pr(A \geq 1)$ 1 mark

$\Pr(A \geq 1) = \Pr(A = 1) + \Pr(A = 2)$
$2.5(0.2) + 5(0.2)^2 = 0.7$ [A1]

c. Calculated the expected number of CAS's that would go missing in one week. 2 marks

$E(A) = [(1 \times 0.5) + (2 \times 0.2)]$ [M1]
$= 0.9$ [A1]

The trend has gotten so bad that the SCHS Maths department is forced to negotiate a CAS replacement program with Officeworks. Each week, the Maths department will place an order with Officeworks. The total cost to replace CAS's per week can be calculated using \$180 for each CAS and \$55 as a flat cost in administration and logistic costs.

d. Using linear transformation and your answer in (c), calculate the expected costs, in dollars, of replacing CAS's each week. 2 marks

$E(180A + 55)$
$= 180 \times E(A) + 55$ [M1]
$= \$217$ [A1]

e. Given that $E[(180A + 55)^2] = 62965$, calculate the standard deviation of the costs of replacing CAS's each week. 2 marks

$Var(A) = E[(180A + 55)^2] - [E(180A + 55)]^2$
$= 62965 - (217)^2$
$= 15876$ [M1]
$SD(A) = \sqrt{15876}$
$= 126$ [A1]

Question 2 (9 marks):
In an isolated incident, Ricardo got his CAS stolen when he left it unattended in a breakout room in the G16-21 area afterschool one day. He found it later being sold on Gumtree.

Chelsea, from his homegroup, thinks she saw who walked into the breakout room. However, she was sitting quite far away, and her accusations are flimsy at best. To verify her claim, Mr. Mott tested her eyesight, by assessing her ability to identify a student from the claimed distance away.

The probability that she would correctly identify a student is 0.8. In the test to identify 8 students from distance, B is the random variable following a binomial distribution for the number of students she identifies correctly.

a. What is the expected number of students she will correctly identify? 1 mark

$B \sim Bi\ (8, 0.8)$
$E(B) = 8 \times 0.8$

$= 6.4$ [A1]

b. What is the probability that she correctly identified exactly 7 students correctly? Express your answer to four decimal places. 1 mark

$\Pr(B = 7) = 0.3355$ [A1]

c. What is the probability that she correctly identified 7 or more students correctly? Express your answer to four decimal places. 1 mark

$\Pr(B \geq 7) = 0.5033$ [A1]

d. What is the probability she identified exactly 7 students correctly, given that she correctly identified at least four of them? Express your answer to four decimal places. 2 marks

$\Pr(B = 7|B \geq 4)$

$= \frac{\Pr(B=7)}{\Pr(B\geq 4)}$

$= \frac{0.335544}{0.989594}$

$= 0.3391$ [A1]

e. What is the probability she identified exactly 7 students correctly, given that she correctly identified the first two? Express your answer to four decimal places. 2 marks

$\Pr(B = 7|first\ two\ right) = \frac{\Pr(B=7 \cap 1st\ two)}{\Pr(1st\ two)}$

$= \frac{0.251658}{0.64}$ [M1]

$= 0.3932$ [A1]

f. What is the minimum number of people in the test to ensure that the probability of Chelsea identifying at least seven correctly is at least 0.46? 2 marks

$B_2 \sim Bi(n, 0.8)$

$\Pr(B \geq 7) > 0.46$ [M1]
$n = 8$ [A1]

Question 3 (9 marks):
Mr. Chau spends most of his Thursday afterschool in the G16-21 area for Maths Help. While the crime occurred on Thursday, it did not occur while Mr. Chau was at Maths Help. Mr. Chau uses his probability distribution of how long he stays at Maths Help to figure out the timeframe in which the theft took place.

The random variable (C) for the amount of time Mr. Chau spends at Maths help (in minutes) can be described by the continuous probability density function:

$$f(t) = \begin{cases} -\frac{1}{80000}t(t-80), & 0 < t \leq 40 \\ mt + c, & 40 < t \leq 80 \\ 0, & otherwise \end{cases}$$

Where m is a negative rational number, and c is a real number.

a. Given that $f(t)$ is continuous and that $\int_0^{80} f(t)dt = 1$, write down **two linear equations** to solve for m and c. Do not solve these equations. 2 marks

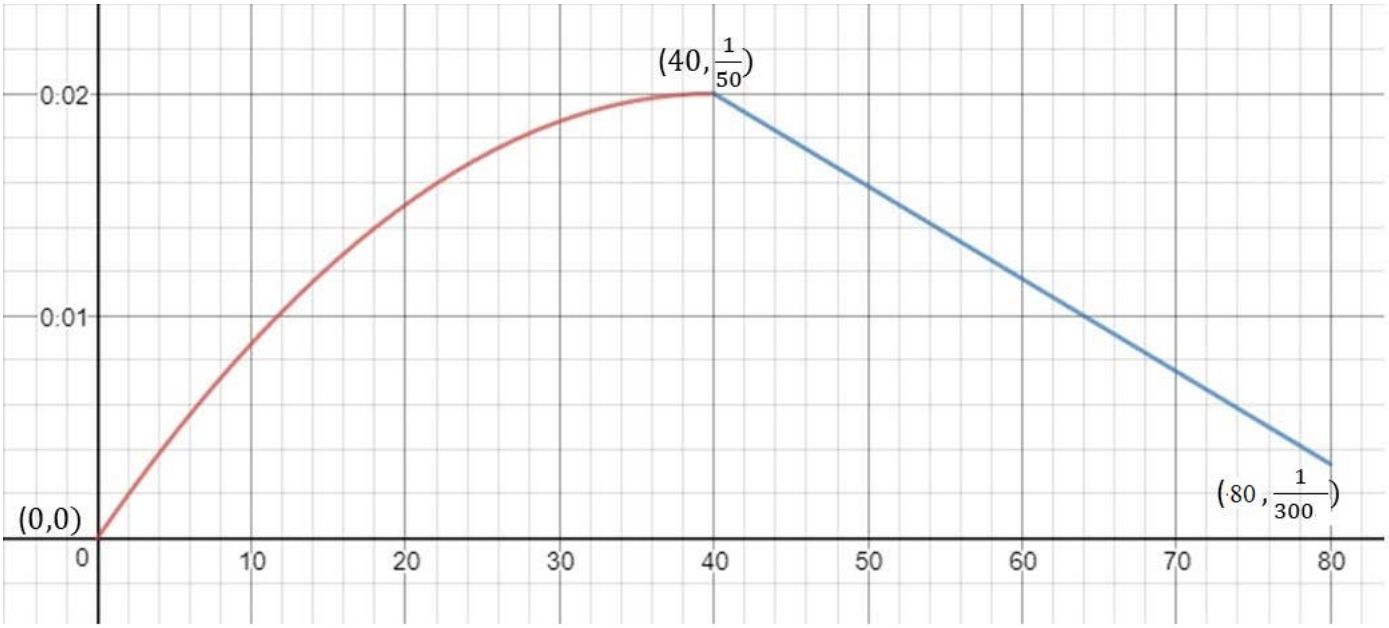
$f(40) = \frac{1}{50}$
$\frac{1}{50} = 40m + c$ [A1]
$\int_0^{80} f(t)dt = 1$
$36000m = -600c + 7$ [A1]

Solving those equations give $m = -\frac{1}{2400}$ and $c = \frac{11}{300}$.

Using the values of m and c above, the random variable (C) for the amount of time Mr. Chau spends at Maths help (in minutes) can hence be described by the continuous probability density function:

$$f(t) = \begin{cases} -\frac{1}{80000}t(t-80), & 0 < t \leq 40 \\ -\frac{1}{2400}t + \frac{11}{300}, & 40 < t \leq 80 \\ 0, & otherwise \end{cases}$$

b. On the grid below, sketch the graph of the continuous probability distribution. Label **in exact coordinates** the end points, and the point where the two pieces of the hybrid function meet. Be aware of the inclusion of end points. 2 marks



$\frac{1}{2}$ mark each for labelling each coordinate f(40) and f(80)
$\frac{1}{2}$ mark for correct shape for both graphs
$\frac{1}{2}$ mark for correct open/closed end points

c. Calculate the probability that Mr. Chau stays for Maths Help for at most 30 minutes on any given Thursday. 1 mark

$\int_0^{30} f(t)\,dt$
$= \frac{27}{80}$ [A1]

d. Calculate the probability that Mr. Chau stays for Maths Help, on a Thursday, for at most 30 minutes given that he stayed for at least 20minutes. 2 marks

$\Pr(C < 30 \mid C > 20) = \frac{\Pr(20<C<30)}{\Pr(C>20)}$
$= \frac{\frac{41}{240}}{\frac{5}{6}}$ [M1]
$= \frac{41}{200}$ [A1]

The crime took place after the median time Mr. Chau is at Maths help. Maths help starts at 3.15pm.

e. i) Write an expression to calculate the median time which Mr. Chau stays for maths help. 1 mark

$\int_0^m f(t)\,dt = 0.5$ [A1]

e. ii) Hence, using your answer from e(i), state the **time** after which the crime occurred. Express your answer to the nearest minute. 1 mark

$m = 38.33$
$3.15 + 38 = 3.53pm$ [A1]

Question 4 (7 marks):
Mr. Lam happened to be recording one of his lessons in a nearby room and may have caught an unclear glimpse of the perpetrator (student who committed the crime) in the background! To verify Chelsea's accusations, investigators used the height of the perpetrator to narrow down the number of suspects.

The height of Suzanne Cory students, described by random variable D such that it is normally distributed with a mean 160cm and standard deviation of 6cm.

$$D \sim N(160, 6^2)$$

a. Calculate the probability that the perpetrator is between 155cm and 170cm. Express your answer to four decimal places. 1 mark

$\Pr(155 \leq D \leq 170) = 0.7499$ [A1]

One of the suspects, Nimeth, has a height of 168 cm.
b. Calculate the z-score of Nimeth's height. 1 mark

$z = \frac{168-160}{6}$
$z = \frac{4}{3}$ [A1]

Based on the unclear video footage, the perpetrator looked to be within 1 standard deviation of the mean.

c. Using your answer in the part (b), explain, in words why Nimeth would be innocent. 1 mark

A z-score of $\frac{4}{3}$ indicates that Nimeth is $1.\dot{3}$ standard deviations from the mean,which is outside the range of required standard deviation. [A1]

d. Calculate the probability that the perpetrator is between 162 and 170, given that he is within 1 standard deviation. Express your answers to four decimal places. 2 marks

$\Pr(162 \leq D \leq 170 \mid 154 \leq D \leq 166)$
$= \frac{\Pr(162 \leq D \leq 166)}{\Pr(154 \leq D \leq 166)}$
$= \frac{0.210786}{0.682689}$ [M1]
$= 0.3088$ [A1]

We went about this all wrong! In the video, we see the perpetrator is a male wearing a year 12 jacket! Therefore, we should work out the heights based on the mean and standard deviation of the year 12 male cohort.

e. Given that 2.275% of the year 12s boys have a height above 179cm, and 25.249% have a height below 167cm, calculate the mean and standard deviation. Express your answers to one decimal place. 2 marks

At 97.725%, $z = 2$
At 25.249%, $z = -0.666675$
$\frac{179-\mu}{\sigma} = 2$
$\frac{167-\mu}{\sigma} = -0.666675$ [M1]
$\mu = 170.0\text{cm}, \sigma = 4.5\text{cm}$ [A1]

END OF SAC 3: Part 2