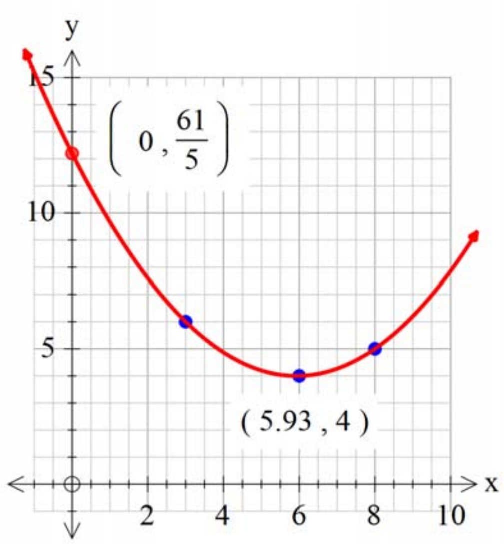
2018 SAC 1 - PREP 1 - solutions

John is a robotics expert. He wishes to create a robot that will follow a particular path. His success will be measured by how close the robot can follow certain criteria. If the robot veers too far from a chosen path it will be deemed unsuccessful. The success of the project will depend on the ability of the robot to determine a suitable curve.

John marks out the experiment area as seen below. His robot will need to pass over the points identified.



$(3,6),\ (6,4)\ \textit{and}\ \ (8,5)$

$6 = 9a + 3b + c$

$4 = 36a + 6b + c$

$5 = 64a + 8b + c$

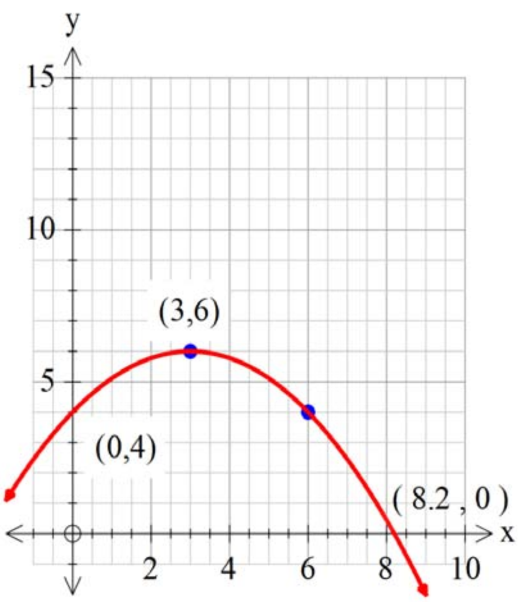
CAS

$a = \dfrac{7}{30},\ b = \dfrac{-83}{30}\ \textit{and}\ \ c = \dfrac{61}{5}$

$y = \dfrac{7}{30}x^2 - \dfrac{83}{30}x + \dfrac{61}{5}$

a) Determine an equation of a parabola that passes through these two points and one other point chosen by you. Sketch the parabola on the axes above.

b) Determine an equation of a parabola that passes through these two points where the turning point occurs at (3,6) and the curve passes through (6,4). Sketch on the axes below.



$(3,6),\ (6,4)$

$y = a(x-3)^2 + 6$

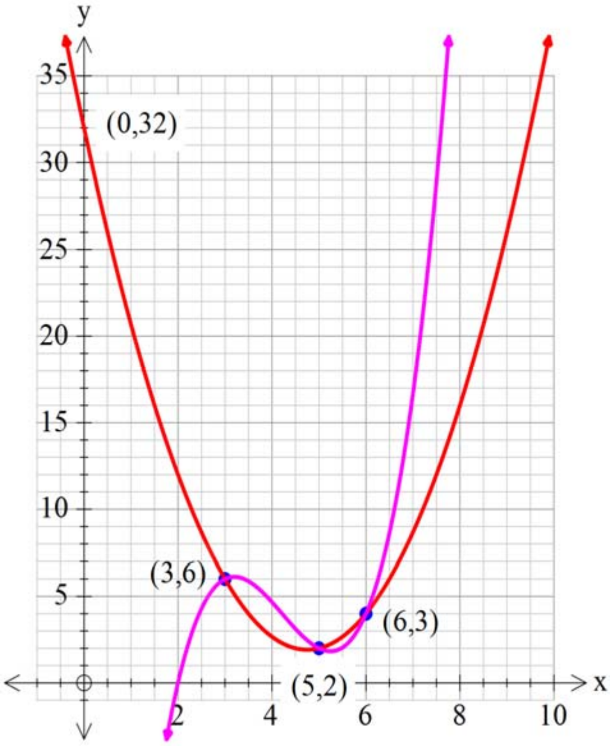
$4 = a(6-3)^2 + 6$

$-2 = 9a$

$a = \dfrac{-2}{9}$

$y = \dfrac{-2}{9}(x-3)^2 + 6$

c) Determine an equation of a parabola that passes through these two points and the point (5,2). Sketch.



$(3,6),\ (6,4)\ and\ (5,2)$

$6 = 9a + 3b + c$

$4 = 36a + 6b + c$

$2 = 25a + 5b + c$

CAS

$a = \frac{4}{3},\ b = \frac{-38}{3}\ and\ c = 32$

$y = \frac{4}{3}x^2 - \frac{38}{3}x + 32$

d) John decides to make the robot's path more challenging by following a cubic rule. He knows that he will need to specify four points for the robot to calculate the path. Explain.

For a cubic there are four unknowns, {a,b,c,d} to determine their value we need to solve four simultaneous equations in four unknowns.

e) Determine the rule for a cubic function ($y = ax^3 + bx^2 + cx + d$ that passes through (3,6), (6,4) and (5,2) where $a = 1$. Sketch the result on the axes above in **part c)**.

$(3,6),\ (6,4)\ and\ (5,2)$

$6 = 27 + 9b + 3c + d$

$4 = 216 + 36b + 6c + d$

$2 = 125 + 25b + 5c + d$

CAS

$b = \frac{-38}{3},\ c = \frac{151}{3}\ and\ d = -58$

$y = x^3 - \frac{38}{3}x^2 + \frac{151}{3}x - 58$

f) Compare and contrast the quadratic found in **part c)** and the cubic found in **part d)**

The two curves do pass through the three required points.

For a domain of $x \in [3,6]$ the curves have a similar position on the axes.

Outside this domain the curves diverge from each other.

Both curves have a local minimum at approximately (5,2)

Parabola lies below the cubic for $x \in (3,5)$ and above the cubic for $x \in (5,6)$

g) Modify the rule for the cubic model so that it more closely matches the quadratic function found in **part c)**. (NOTE: it must pass through the three points but *a* does not equal one).

There are many ways to approach this question. One way is to choose another point to "tighten" the fit.

Select a point on the parabola – say $\left(4, \frac{8}{3}\right)$**.**

$(3,6),\ (6,4), (5,2)\ and\ \left(4, \frac{8}{3}\right)$

$6 = 27a + 9b + 3c + d$

$4 = 216a + 36b + 6c + d$

$2 = 125a + 25b + 5c + d$

$3 = 64a + 16b + 4c + d$

CAS

$a = \frac{1}{6},\ b = -1,\ c = \frac{-13}{6}\ and\ \ d = 17$

$y = \frac{1}{6}x^3 - x^2 - \frac{13}{6}x + 17$