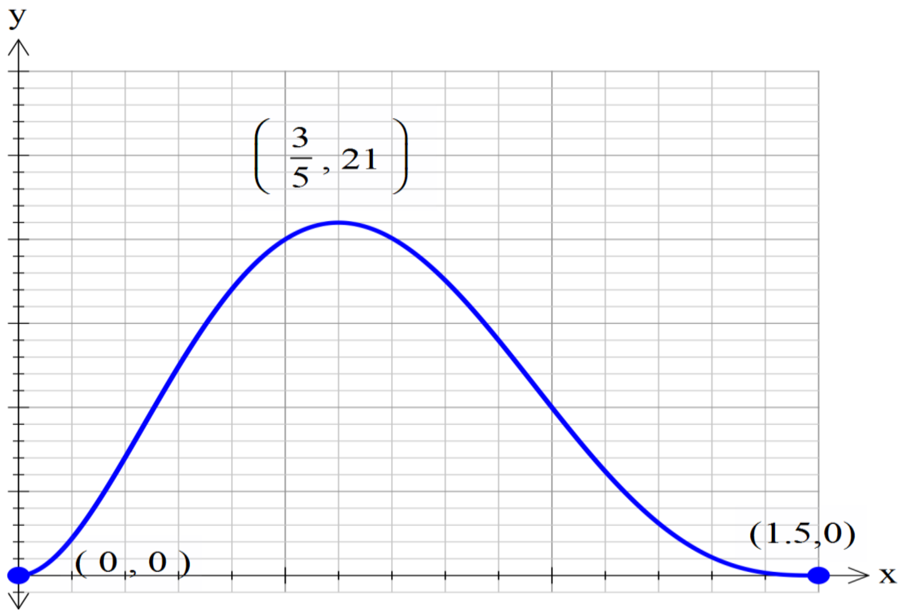
2018 SAC PREP 3 - Solutions

John investigates a path of the form $f(x) = -5px^2(2x-3)^3, x \in \left[0, \frac{3}{2}\right]$.

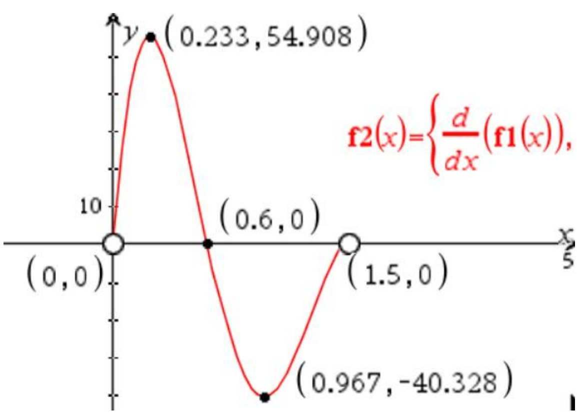
a) Sketch a graph of the robot's path when $p = 2$.



b) Find the average rate of change between $x = 0$ and $x = 1$.

$$AROC = \frac{f(1) - f(0)}{1 - 0}$$

c) Sketch a graph of the derivative function $f'(x)$ when $p = 2$, for a suitable domain. Turning points can be shown to three decimal places. State the domain.



Domain $0 < x < \frac{3}{2}$

d) Find the coordinates, correct to 3 decimal places, of the point on the robot's path where the gradient is the same as the average rate of change found in **part b)**.

$$10 = f'(x)$$
$$10 = -20x(2x-3)^2(5x-3)$$
$$x = 0.020 \ or \ x = 0.550$$
$$(0.020, 0.100) \ or \ (0.550, 20.745)$$

$\frac{d}{dx}(f1(x))$

$\left\{-20 \cdot x \cdot (2 \cdot x-3)^2 \cdot (5 \cdot x-3), 0<x<\frac{3}{2}\right.$

$\text{solve}\left(-20 \cdot x \cdot (2 \cdot x-3)^2 \cdot (5 \cdot x-3)=10, x\right)$ $\quad x=0.019658 \text{ or } x=0.54964$

$f1(x)|x=\{0.019657641277752, 0.5496400390$ ▸ $\quad \{0.100286, 20.7449\}$

e) Find the angle θ from the positive direction of the x-axis to the tangent to the graph of f at $x = 1$, measured in an anti-clockwise direction.

$$f'(1) = -20(1)(2(1)-3)^2(5(1)-3)$$
$$f'(1) = -40$$
$$\theta = \tan^{-1}(-40)$$
$$\theta = 91.43^o$$

$(\tan^{-1}(-40))$ ▸DD $\quad -88.5679038158°$

$180+-88.56790381583 \quad 91.4320961842$

John wants the robot's path to be smooth and has decided that he wants the angle θ from the positive direction of the x-axis to the tangent to the graph of f to always be less than 75^{o}.

f) Use a suitable method to determine if when $p = 2$ this requirement is met.

$$\theta = 75^o$$
$$m = \tan(75^o)$$
$$m = 3.732$$

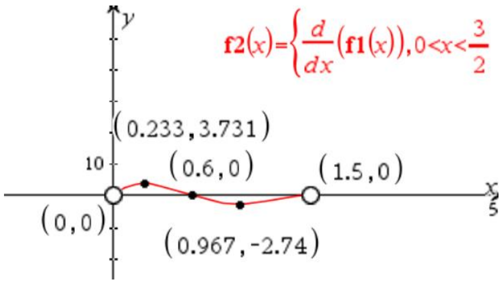
By considering the graph drawn in part c) we can see that the condition is not met as a maximum gradient of 3.732 is allowed. The gradient of the curve exceeds this value.

f) If the condition is not met, determine a suitable value of p that would achieve this requirement.

Maximum gradient is 54.908 we need to dilate this graph by a suitable factor to ensure the gradient is smaller than 3.732.

$$54.908 \times k = 3.73205$$
$$k = 0.068$$

$$p = 0.068 \times 2 = 0.1359(2dp)$$



During some competitions more than one robot can be active in the demonstration area. One of the other robots brushes past John's robot (tangent) when $x=1$. Return to the curve when $p=2$

g) Determine the equation of the tangent at this point.

$\frac{d}{dx}(f1(x))\|x=1$	-40
$f1(1)$	10
$\text{solve}(y-10=-40\cdot(x-1),y)$	$y=50-40\cdot x$

$$f(1)=10$$
$$f'(1)=-40$$
$$y-10=-40(x-1)$$
$$y=50-40x$$

If worth one mark:

$y=\text{tangentLine}(f1(x),x,1)$	$y=50-40\cdot x$

h) An out of control robot, travelling in a straight line hits John's robot. Their paths are at right angles to each other. If they collide when $x=1$, determine the equation of the other robot's path.

$\text{solve}\left(y-10=\frac{1}{40}\cdot(x-1),y\right)$	$y=\frac{x}{40}+\frac{399}{40}$
$\text{normalLine}(f1(x),x,1)$	$\frac{x}{40}+\frac{399}{40}$

i) Determine the equation of the tangent when $x=a$. Check your result on your CAS.

$$f(a)=-10a^2(2a-3)^3$$
$$f'(a)=-20a(2a-3)^2(5a-3)$$
$$y+10a^2(2a-3)^3=-20a(2a-3)^2(5a-3)(x-a)$$
$$y=-20a(2a-3)^2(5a-3)(x-a)-10a^2(2a-3)^3$$