## 2018 SAC 2 - PREP 3

## **Ouestion 1**

Area under a graph can be estimated using rectangles of varying widths. Use this technique and the indicated number of rectangles to approximate the shaded area.

Find the area under the curve  $y = x^2$  between x = -3 and x = 3 using:

a) six left hand rectangles

ii) 12 right hand rectangles

iii) Use calculus to find the exact area and explain why the above approximations are either too low or too high.

(2 + 2 + 2 = 6 marks)

## **Ouestion 2**

SAFE Constructions is employed to build a new frontal coating on an old rollercoaster at Luna park. The section that needs work is modelled by part of the curve  $y = 4x^3 - 30x^2 + 56x + 2$ , where y is the height, in metres, of the rollercoaster above the ground, and x is the horizontal distance, in metres, from some fixed point O.

SAFE Constructions is planning to face the required area with sheets of rectangular, coloured iron that start at the ground and finish in height somewhere approximating the top of the rollercoaster. A problem they need to solve is to make sure that the sharp top corners of the iron sheets do not interfere with the running of the rollercoaster.

Initially, the workers of the SAFE Constructions measure up and cut three equally wide rectangular sheets of iron. These are then cut so that the height of the rectangular sheet matches the left-hand end of the curve.

a) Calculate the height of the rollercoaster for  $x \in \{0,1,2,3\}$ .

0)	width 1 metre, for $x \in [0,3]$ and hence calculate the sum of the areas of the rectangular sheets.	
	is a problem with iron jutting into the rollercoaster, so that workers cut the sheets so the softhe rectangular sheets match the right hand height of the curve.  Calculate the sum of the areas of these new right endpoint rectangular sheets.	2 marks at the
	is still a problem with the height of the sheets so the workers decide to use sheets of sn	2 marks
width.	Calculate the left hand endpoint approximate area covered by 6 sheets of 0.5 m width	
e)	Calculate the left hand endpoint approximate area covered by 12 sheets.	2 marks
f)	Write down and evaluate an expression for the left hand endpoint estimate using 15 st	2 marks
g)	Write down and evaluate an expression for the right hand endpoint estimate using 15	2 marks

h)	Comment on the reliability of the left and right hand estimates.
	1 mark
	Constructions employed an engineer to investigate the situation. She concluded that the most nt method of cutting a new frontal coating would be to cut the exact shape from one sheet of
j)	Write down and evaluate the integral statement that calculates the exact area.
	3 marks
Quest	ion 3
	lium is kept at a constant temperature $T$ of $20^{\circ}C$ . A body is placed in this medium. The rature at time $t$ min of this body is given by:
	$T = 40e^{-0.36t} + 20, t \ge 0$
a)	Find the initial temperature of the body.
	1 mark
b)	Sketch the graph of $T$ against $t, t \ge 0$ .
	2 marks
c)	Find the rate of change of temperature with respect to time (in terms of $t$ ).

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d)	Find:

- (i) The average rate of change between t = 3 and t = 5 minutes.
- (ii) Find the instantaneous rate of change at t = 5 minutes.
- (iii) Find the average value of the temperature of the medium for the first 5 minutes.

2 + 1 = 3 marks