2018 SAC 2 PREP 1

Question 1

A pendulum's motion relative to an equilibrium point can be modelled by the rule

$$f(x) = 3e^{-x}\cos(x), \text{ for } 0 \leq x \leq \pi.$$

Where f is the displacement in centimetres from the equilibrium point and x is measured in seconds.

a) Using algebra, find any axial intercepts.

3 marks

b) Use calculus to differentiate $f(x) = 3e^{-x}\cos(x)$ and hence, find any turning points.

3 marks

c) Sketch the graph of the pendulum's motion. labelling intercepts, turning points and end points.

3 marks

d) Determine the greatest distance the end of the pendulum travels before changing direction. After how long does this occur?

2 marks

e) Find the average rate of change of the function between $x = 0$ and $x = \frac{3\pi}{4}$. Express your answer, correct to two decimal places.

2 marks

f) Find the instantaneous velocity when $x = \frac{\pi}{2}$ as an exact value.

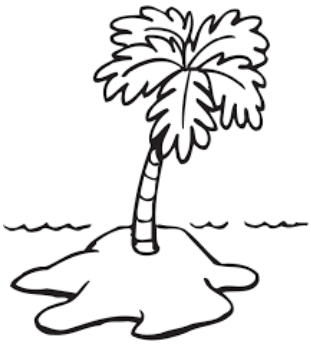
1 mark

g) Determine the period of the pendulum's motion.

1 mark

Question 2

Tom is marooned on an island in a region that has extreme tides. He is able to accurately measure the depth of the water around the island. At midday the depth of the water is at a maximum depth of 4.8 metres. He has also found that 24 hours separates the high tides. At low tide the depth of the water is 40 cm. The depth of the water can be modelled by the function of the form $h(t) = a\cos(bt) + c, t \geq 0$, where h is the depth of the water at time t hours after 12 noon.



a) Determine the value of a, b and c. 3 marks

A reef joins the island Tom is on, to a neighbouring island. The depth of the water on the reef is still modelled by the function given above. Tom reasons that he will have a greater chance of being rescued if he can make it to the other island. Being unable to swim, he needs the water to be no deeper than 1.5 metres in order to complete the crossing.

b) Sketch the first period of the graph of h against t, indicating on your graph the times available to him to complete the crossing.

4 marks

d) How much time does he have to complete the crossing.

1 mark

e) If Tom is afraid of sharks and will only cross if the depth of the water is less than 1 metre, How will this affect the time he has to make the crossing?

3 marks

To his horror, Tom estimates that it will take him more than 10 hours to complete the crossing. He decides to build a fire at the high point of the island to attract a passing ship. Tom calculates that the type of ship that passes the island needs a depth of at least 4.5 metres so that it does not run aground.

f) Between what times during the day can a ship approach the island?

2 marks

g) If the ship must travel 10 km to reach the island, what speed in km/hr must the ship average to save Tom?

2 marks