2018 SAC 2 PREP 4

Question 1

A particle moves in a straight line so that its position x centimetres relative to O at time t seconds is given by $x = t^2 - 7t + 6, t \geq 0$.

Find:

a) the initial position of the particle

b) the position of the particle after 6 seconds.

c) the distance travelled in the first 6 seconds.

d) the average speed over the first 6 seconds.

e) the average velocity over the first 6 seconds.

f) the particle's initial velocity and acceleration with appropriate units.

g) when and where the particle is at rest.

h) sketch a graph of:

i) position against time

ii) velocity against time

(1 + 1 + 2 + 2 + 2 + 2 + 3 + 2 + 2 = 18 marks)

Question 2

Dr CAS is running late for school. He parks his car and starts running along the footpath towards the school gate. All of a sudden he realises he has left his CAS calculator in his car. He turns around and runs back in the direction of the car. On his way back he remembers that he packed his CAS calculator in his bag the night before. He turns around and starts running in the direction of the gate. His position relative to the gate is given by $x = \frac{1}{10}\left(\frac{t^4}{4} - \frac{14t^3}{3} + \frac{45t^2}{2} - 150\right)$.

a) Determine how far from the gate Dr CAS parked his car.

b) After how many seconds did Dr CAS turn around and start running back to his car?

c) How far has Dr CAS run in the direction of the gate before he turns around for the first time?

Dr CAS hears the school bell ring 10 seconds after he leaves his car.

d) How far is he from the school gate?

One of Dr CAS' students loves to race him to the school gate. He meets him at his parked car and then they race to the school gate. The position of the student at any time is given by:

$$x_S = \begin{cases} 2.1t - 15, & 0 \le t \le 6 \\ 0.096(t-6)^2 - 2.4, & 6 < t < Finish \end{cases}$$

e) Find the time(s) and position(s) where they are the same distance from the gate.

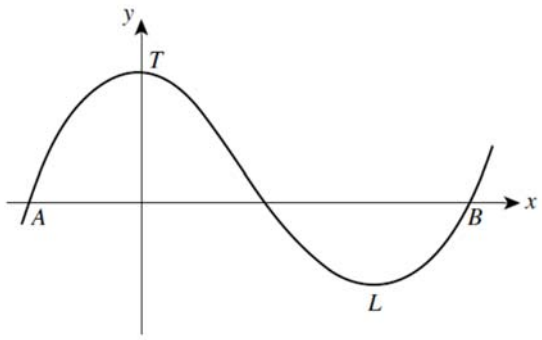
f) Determine who reaches the school gate first and at what time this occurs.

g) Determine if there are any times when Dr CAS and the student are running at the same velocity.

(1 + 2 + 3 + 1 + 3 + 2 + 3 = 15 marks)

Question 3

Phil Hotham is a construction engineer working to build a new ski resort at Fool's Creek. Part of the plan of the main ski run on the mountain is shown in the diagram below.



This section of the mountain can be mapped by the curve with equation $f(x) = \frac{x^3}{480} - \frac{7x^2}{48} + 75$ from A to B. The x-axis represents the level at which the village will be built. T is the highest point of the ski run and L is the lowest point. All distances are in metres.

b. Find the coordinates of A and B.

2 marks

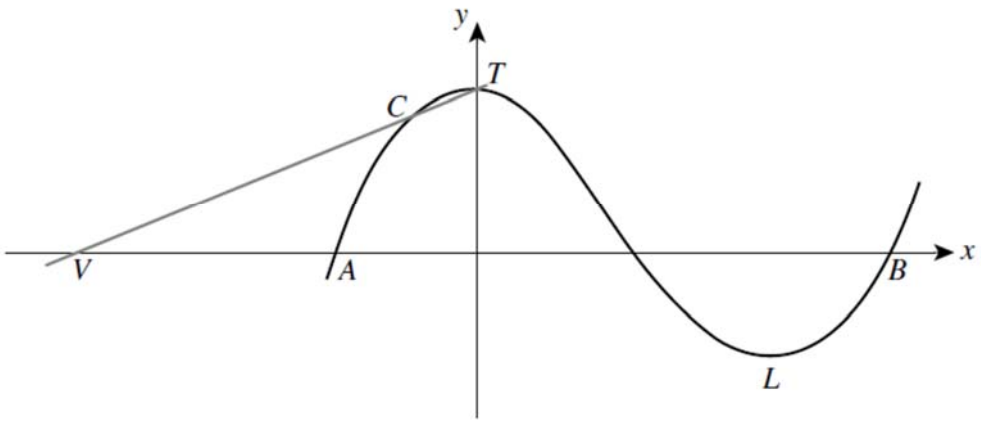
b. Find the coordinates of T.

1 mark

c. Find the average gradient of the ski run from T to L.

2 marks

Phil is planning a chairlift to take the skiers to the top of the mountain from the village. He plans to build the village at the point V, which is 40 m from A. The path of the chairlift is shown on the diagram below from V to T.



d. Find the equation of the line which models the path of the chairlift.

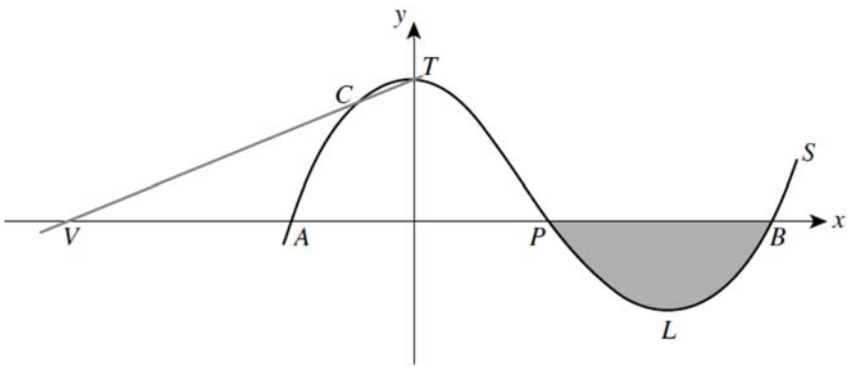
1 mark

Phil has found the chairlift will reach the mountainside at the point *C* before it gets to the top of the mountain. He decides to remove the part of the mountain that is obstructing the chairlift.

e. Determine the cost of removing this soil and rock if it costs \$40/m^2.

3 marks

Phil is considering adding a water slide to be built down the other side of the mountain, beginning at point *S* and entering the pond at point *B.* Safety regulations require that the average gradient of the slide be no more than one third of the depth of the water at the deepest part of the pond.



f. Find the coordinates of *S*, the highest point on the mountain that the water slide could start from and still satisfy the safety regulations. Give your answer to the nearest centimetre.

4 marks