2018 SAC PREP 1

Question 1

Brett and Dora each subscribe to a particular website.
The time X, in seconds, it takes Brett to log-in to this website has a probability density function given by

$$f(x) = \begin{cases} 0 & x < 0 \\ 0.3125 & 0 \le x \le 3 \\ \dfrac{5}{16}e^{-5(x-3)} & x > 3 \end{cases}$$

The time Y, in seconds, it takes Dora to log-in to the same website is normally distributed with a mean of 2 and a standard deviation of 0.6.

a. Find the probability of it taking less than 1 second for Dora to log in. Express your answer, correct to four decimal places.

1 mark

b. Sixty percent of the time, Dora can log-in to the website in under p seconds. Find the value of p correct to 4 decimal places. Include in your answer a suitable diagram.

2 marks

c. Sketch the continuous probability for Brett's login times on a suitably labelled axis.

d. Find the probability that it takes Brett between 2 and 4 seconds to log in. Express your answer, correct to four decimal places.

3 marks

e.   Find the variance of

    i.   *Y*

    ii.   *X*, express your answer, correct to four decimal places.

$$1+3=4 \text{ marks}$$

f.   Six of Dora's log-in times are randomly selected to check the efficiency of her computer. What is the probability, correct to 4 decimal places that:

    i.   exactly one of these log-in times took less than 2 seconds?

    ii.   at least one of these log-in times took less than 2 seconds?

$$2+1=3 \text{ marks}$$

Brett and Dora's subscriptions to the website expire after a certain, large number of log-ins have been made. Both their subscriptions have just expired. The log-in times for both Brett and Dora during their subscriptions were analysed and a randomly selected log-in time was found to be less than 1 second.

g.   **i.**   Use a tree diagram to represent this situation. Consider who made the login and the length of time it took to connect.

    ii.   Hence, find the probability correct to three decimal places that this log-in was one that Dora made given that it was found to have taken less than 1 second.

$$1 + 3 = 4 \text{ marks}$$

Total 16 marks

Question 2

At a seminar for Mathematical Methods, teachers are quizzed on the other mathematics subjects that they teach. It is found that 60% teach Further Mathematics, 10% teach both Specialist and Further and 22% teach no other mathematics aside from Mathematical Methods.

a.   Use a suitable probability technique to display this information.

b.   Hence find the probability that a teacher:
   i.   teaches Further Mathematics but not Specialist Mathematics.

   ii.   does not teach Specialist Mathematics.

   iii.   teaches Specialist Mathematics if it is known they do not teach Further Mathematics.

$2 + 1 + 1 + 2 = 6$ marks

c.   Explain whether the events 'teaches Further Mathematics' and 'teaches Specialist Mathematics' are independent.

2 marks

d.   Explain whether the events 'teaches Further Mathematics' and 'teaches Specialist Mathematics' are mutually exclusive.

2 marks

At a three-day conference, there are two options for lunch: vegetarian and non-vegetarian. John, a mathematics teacher finds that if orders a vegetarian lunch one day, the probability that he orders the non-vegetarian option the following day is 0.75. However, if he orders non-vegetarian one day, the probability that he orders vegetarian the following day is 0.55. John chose the vegetarian option on the first day (Monday) of the conference.

e.   Construct a tree diagram to represent John's choices for the three days of the conference.

3 marks

f.   Find the probability that Jamie will choose the non-vegetarian option at least once over the next two days. Give answer correct to four decimal places.

2 marks

g.   Find the probability that John chooses the vegetarian option on the last day of the conference given that he chose the non-vegetarian option at least once over the next two days.

2 marks

John has found over time that at a three-day conference the number of days he chooses a vegetarian option is a discrete random variable, X. The discrete random variable has probability distribution

x	0	1	2	3
$\Pr(X=x)$	$2p$	$2p^2$	p^2+p	$2p^2+p$

h.   Find the value of p.

2 marks

i.   Find the expected number of vegetarian meals that John orders at the three-day conference.

2 marks

j.   Find the probability that John will have at least 2 vegetarian meals at the conference.

2 marks

k. Determine a 95% confidence interval for the number of vegetarian lunches ordered at a three-day conference.

<div align="right">3 marks</div>

John plans to attend 3 conferences in 2017.

l. Find the probability that he will order the same number of vegetarian lunches at the three conferences.

<div align="right">2 marks</div>

m. Find the probability that he will order a total of 2 vegetarian lunches over the three conferences.

<div align="right">2 marks</div>