## 2018 SAC 3 PREP 2

## **Question 1**

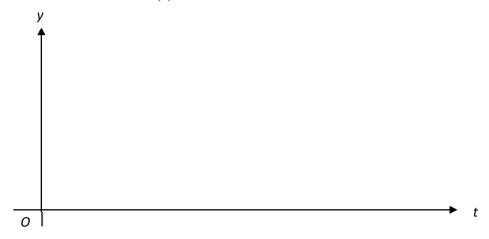
At a cinema, the time T minutes, that customers have to wait in order to buy their tickets has a continuous probability density function given by

$$f(t) = \begin{cases} bt^2 & \text{for } 0 \le t \le 4\\ c(8-t) & \text{for } 4 < t \le 8\\ 0 & \text{otherwise} \end{cases}$$

**a.** Show that  $b = \frac{3}{160}$  and  $c = \frac{3}{40}$ .

3 marks

**b.** Sketch the graph of y = f(t) on the axes provided, clearly labelling the scale.



3 mark

c. Use calculus to find the probability that a customer waits more than six minutes to buy a ticket.

d.	Find the mean time that customers have to wait in order to buy their tickets, give your answer, correct to one decimal place.	
e.	2 mar Find the median time, correct to two decimal places, that customers have to wait in order to buy their tickets.	ks
<b>f.</b> S	2 mar State one reason why the mean and the median have similar values.	ks
	2 mar	ks
	nning times of movies shown at the cinema are normally distributed with a mean of 94 es, with a standard deviation of 10 minutes.	
g.	Find the probability that the running time of a movie is more than 109 minutes, give your answ correct to three decimal places.	er
	1 ma	rk
h.	A certain cinema complex has four different movies playing. Find the probability that at least to of the movies have a running time of more than 109 minutes, give your answer correct to three decimal places.	WО
		1
	2 mar	ks
Pete g	oes to the movies once every week. He likes to see only action or comedy movies.	
0.45, v	ees an action movie one week, the probability that he sees an action movie the following week is while if he sees a comedy one week, the probability that he sees an movie the following week is 0.35. Suppose he has just seen an action movie.	;
i.	What is the probability that of the next three movies he sees, exactly two are comedies, give yo answer correct to three decimal places.	ur
	2 mar	ks

## **Question 2**

At George's garage, the time, X hours, taken to service a car follows the probability density function f where

$$f(x) = \begin{cases} 0 & x < 2\\ \frac{(x-5)^2 + 1}{12} & 2 \le x \le 5\\ 0 & x > 5 \end{cases}$$

a	Find the mean	time taken to	service a	car at Georg	e's garage
a.	Tillu tile illean	unic taken te	service a	cai ai Georg	e s garage

1 mark

Last week there were 40 cars serviced at George's garage. Andrew's car was among the fastest 25% of cars to be serviced there last week.

**b.** What was the maximum time that Andrew's car could have taken to be serviced? Express your answer in hours, correct to 3 decimal places.

2 marks

The diameters, in mm, of a consignment of 700 cylindrical filters used at the garage are normally distributed with a mean of 24mm and a standard deviation of 0.3mm.

George has found that the widest 20% of these filters cannot be used in Mini cars.

**c.** What is the maximum diameter of a filter that can be used in a Mini car? Express your answer in mm correct to 2 decimal places. Use a suitable diagram in your solution.

2 marks

Filters with a diameter of less than 23.5 mm will fit in any car.

**d.** How many filters in this consignment will fit in any car? Express the answer correct to the nearest whole number.

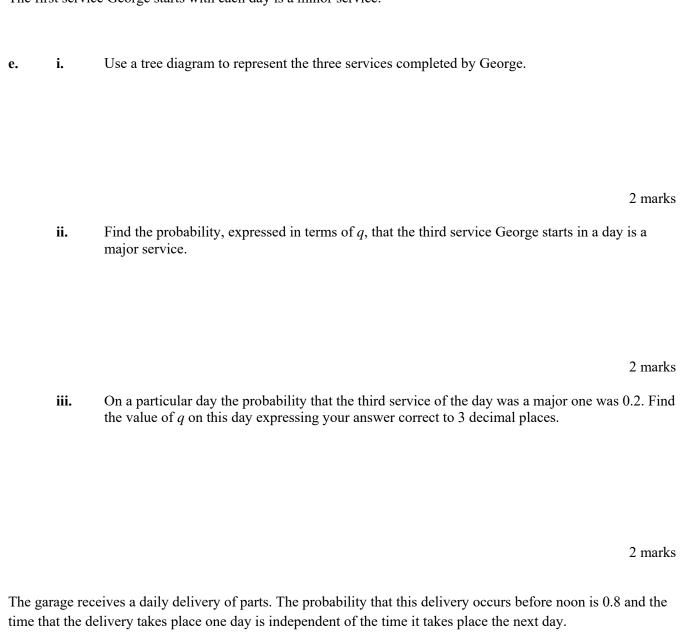
2 marks

The services George performs at his garage are classified as major or minor services depending on the number of kilometres a car has done.

George starts each service and often hands over to another mechanic to finish a service. He has found over time that if a service he starts is major then the probability that the next service he starts is also major is 0.4.

If a service he starts is minor then the probability that the next service he starts is minor is q where 0 < q < 1 and q can vary from day to day.

The first service George starts with each day is a minor service.



**f.** i. The probability that over an n day period the delivery occurred each day before noon is 0.32768. Find the value of n.

1 mark

	ii.	Find the probability that over a 20 day period, the delivery arrives before noon on more than 16 days given that on at least half of the 20 days it arrives before noon. Express your answer correct to 4 decimal places.	
		2 mark	a
		2 mark	5
		is a mail delivery each work day. The probability of the mail being delivered before a certain time of as $\Pr(T < t) = m$ , where $m \in (0,1)$ .	
g.	Find ar	expression that represents the mail being delivered before this certain time on 2 of the five days.	
		2	_
		3 mark	S
h.	Show to	hat the maximum value this probability can take is $0.3456$ . Also find the value of $m$ for this ility.	
		2 mark	S
			,
i.		known that the delivery times are actually normally distributed with mean of midday (12 o'clock) ndard deviation of 1 hour, find the certain time $t$ , correct to the nearest minute.	
		2 mark Total 23 mark	