2018 SAC 3 PREP 1 - SOLUTIONS

Question 1

Brett and Dora each subscribe to a particular website.
The time X, in seconds, it takes Brett to log-in to this website has a probability density function given by

$$f(x)=\begin{cases} 0 & x<0 \\ 0.3125 & 0 \le x \le 3 \\ \dfrac{5}{16}e^{-5(x-3)} & x>3 \end{cases}$$

The time Y, in seconds, it takes Dora to log-in to the same website is normally distributed with a mean of 2 and a standard deviation of 0.6.

a. Find the probability of it taking less than 1 second for Dora to log in. Express your answer, correct to four decimal places.

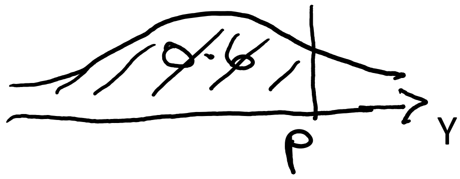
$\Pr(Y<1)=0.0478$

1 mark

b. Sixty percent of the time, Dora can log-in to the website in under p seconds. Find the value of p correct to 4 decimal places. Include in your answer a suitable diagram.

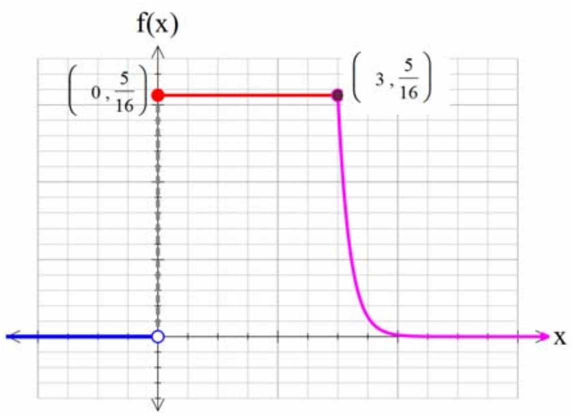
$Y \sim N(2, 0.6^2)$

$\Pr(Y<p)=0.6$
$p=2.1520$



2 marks

c. Sketch the continuous probability for Brett's login times on a suitably labelled axis.



There is a horizontal asymptote at $y=0$ for $x \to \infty$

d. Find the probability that it takes Brett between 2 and 4 seconds to log in. Express your answer, correct to four decimal places.

$\Pr(2<X<4)=\int_2^4 f(x)dx=0.3746$

3 marks

e. Find the variance of

i. Y $var(Y) = 0.6^2 = 0.36$

ii. X, express your answer, correct to four decimal places.

$$\text{var(X)} = \int_0^{\infty} x^2 f(x)dx - \left(\int_0^{\infty} xf(x)dx\right)^2$$

$$var(X) = 3.455 - (1.60625)^2$$
$$var(X) = 0.8750$$

NB: The CAS seems to not like this one so much but you can use nInt(x^{2}f(x),x,-∞,∞)

1 + 3 = 4 marks

f. Six of Dora's log-in times are randomly selected to check the efficiency of her computer. What is the probability, correct to 4 decimal places that:

i. exactly one of these log-in times took less than 2 seconds?
Let D be the number of Dora's login times that is less than 1 second.

$$D \sim Bi(6,0.5)$$
$$\Pr(D = 1) = 0.0938$$

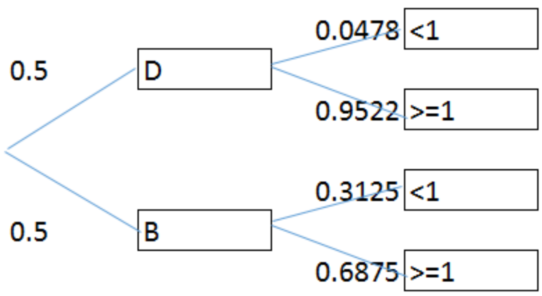
ii. at least one of these log-in times took less than 2 seconds?

$$Pr(D \geq 1) = 0.9844$$

2 + 1 = 3 marks

Brett and Dora's subscriptions to the website expire after a certain, large number of log-ins have been made. Both their subscriptions have just expired. The log-in times for both Brett and Dora during their subscriptions were analysed and a randomly selected log-in time was found to be less than 1 second.

g. **i.** Use a tree diagram to represent this situation**.** Consider who made the login and the length of time it took to connect.



ii. Hence, find the probability correct to three decimal places that this log-in was one that Dora made given that it was found to have taken less than 1 second.

$$\Pr(Dora|time < 1 \text{ second}) = \frac{\Pr(D\cap<1)}{\Pr(<1)}$$

$$\Pr(Dora|time < 1 \text{ second}) = \frac{0.5 \times 0.0478}{0.5 \times 0.0478 + 0.5 \times 0.3125}$$

$$\Pr(Dora|time < 1 \text{ second}) = \frac{0.0239}{0.18015}$$

$$\Pr(Dora|time < 1 \text{ second}) = 0.133$$

1 + 3 = 4 marks

Total 16 marks

Question 2

At a seminar for Mathematical Methods, teachers are quizzed on the other mathematics subjects that they teach. It is found that 60% teach Further Mathematics, 10% teach both Specialist and Further and 22% teach no other mathematics aside from Mathematical Methods.

a. Use a suitable probability technique to display this information.

	F	**F'**	
S	0.1	0.18	0.28
S'	0.5	0.22	0.72
	0.6	0.40	1

NB: A Venn diagram could also be used.

b. Hence find the probability that a teacher:

i. teaches Further Mathematics but not Specialist Mathematics.

$$\Pr(F \cap S') = 0.5$$

ii. does not teach Specialist Mathematics.

$$\Pr(S') = 0.72$$

iii. teaches Specialist Mathematics if it is known they do not teach Further Mathematics.

$$\Pr(S|F') = \frac{0.18}{0.40} = \frac{9}{20} = 0.45$$

2 + 1 + 1 + 2 = 6 marks

c. Explain whether the events 'teaches Further Mathematics' and 'teaches Specialist Mathematics' are independent.

To be independent $\Pr(F \cap S) = \Pr(F) \times \Pr(S)$

$\Pr(F \cap S) = 0.1$ $\Pr(F) \times \Pr(S) = 0.6 \times 0.28 = 0.168$

As $0.1 \neq 0.168$ the events are NOT independent.

2 marks

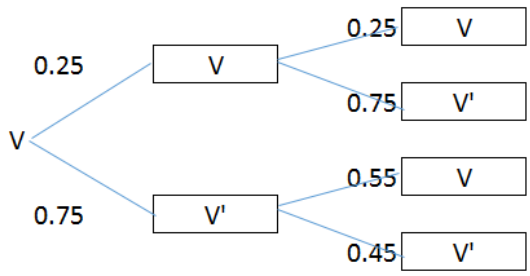
d. Explain whether the events 'teaches Further Mathematics' and 'teaches Specialist Mathematics' are mutually exclusive.

As $\Pr(F \cap S) = 0.1$ the events are not mutually exclusive as this requires $\Pr(F \cap S) = 0$

2 marks

At a three-day conference, there are two options for lunch: vegetarian and non-vegetarian. John, a mathematics teacher finds that if orders a vegetarian lunch one day, the probability that he orders the non-vegetarian option the following day is 0.75. However, if he orders non-vegetarian one day, the probability that he orders vegetarian the following day is 0.55. John chose the vegetarian option on the first day (Monday) of the conference.

e. Construct a tree diagram to represent John's choices for the three days of the conference.



3 marks

f. Find the probability that Jamie will choose the non-vegetarian option at least once over the next two days. Give answer correct to four decimal places.

$$\Pr(at\ least\ one\ vegetarian) = 1 - \Pr(VV)$$
$$\Pr(at\ least\ one\ vegetarian) = 1 - 0.25^2$$
$$\Pr(at\ least\ one\ vegetarian) = 0.9375$$

2 marks

g. Find the probability that John chooses the vegetarian option on the last day of the conference given that he chose the non-vegetarian option at least once over the next two days.

$$\Pr(vegetarian\ on\ Wed|at\ least\ one\ non-vegetarian) = \frac{\Pr(V'V)}{0.9375}$$

$$\Pr(vegetarian\ on\ Wed|at\ least\ one\ non-vegetarian) = \frac{0.75\times0.55}{0.9375} = \frac{11}{25} = 0.44$$

2 marks

John has found over time that at a three-day conference the number of days he chooses a vegetarian option is a discrete random variable, *X*. The discrete random variable has probability distribution

x	0	1	2	3
$\Pr(X=x)$	$2p$	$2p^2$	p^2+p	$2p^2+p$

h. Find the value of *p*.

$$2p + 2p^2 + p^2 + p + 2p^2 + p = 1$$
$$5p^2 + 4p - 1 = 0$$
$$(5p-1)(p+1) = 0$$
$$p \in \left(0, \frac{1}{2}\right) \quad p = \frac{1}{5}$$

2 marks

i. Find the expected number of vegetarian meals that John orders at the three-day conference.

$$E(X) = \sum xPr(X = x) = 0 \times \frac{2}{5} + 1 \times \frac{2}{25} + 2 \times \frac{6}{25} + 3 \times \frac{7}{25} = \frac{7}{5} \ \ or \ \ 1.4$$

2 marks

j. Find the probability that John will have at least 2 vegetarian meals at the conference.

$$\Pr(X \geq 2) = \frac{6}{25} + \frac{7}{25} = \frac{13}{25}$$

2 marks

k. Determine a 95% confidence interval for the number of vegetarian lunches ordered at a three-day conference.

$$var(X) = E(X^2) - \left(E(X)\right)^2$$

$$var(X) = \frac{89}{25} - \left(\frac{7}{5}\right)^2 = \frac{8}{5}$$

$$sd(X) = \sqrt{\frac{8}{5}}$$

Required interval is $(-1.13, 3.93)$ hence $[0,3]$

3 marks

John plans to attend 3 conferences in 2017.

l. Find the probability that he will order the same number of vegetarian lunches at the three conferences.

$$\Pr(same) = \left(\frac{2}{5}\right)^3 + \left(\frac{2}{25}\right)^3 + \left(\frac{6}{25}\right)^3 + \left(\frac{7}{25}\right)^3 = \frac{1567}{15625}$$

2 marks

m. Find the probability that he will order a total of 2 vegetarian lunches over the three conferences.

Consider the possible combinations {1,0,1} or {1,1,0} or {0,1,1} or {2,0,0} or {0,2,0} or {0,0,2}

$$\Pr(same) = 3\left(\frac{2}{25}\right)^2\left(\frac{2}{5}\right) + 3\left(\frac{6}{25}\right)\left(\frac{2}{5}\right)^2 = \frac{384}{3125}$$

2 marks