# 2018 SAC 3 PREP 4 - SOLUTIONS

**Question 1 (21 marks)**
The records of a large hospital show that the healing time for a particular type of surgical incision is a normally distributed random variable with a mean of 11 days and a standard deviation of 2.5 days. Due to a shortage of hospital places, all patients undergoing this surgery are discharged from hospital 13 days after receiving the surgery.

a) Grainger, a hospital supervisor, randomly selects the record of a discharged patient who has undergone this surgery.

i. What is the probability, correct to four decimal places, that the incision had healed by the day the patient was discharged?

$\Pr(X \leq 13) = 0.7881$

(1) X≤13
(1) correct probability (4dp)

ii. Given that, at the time of discharge, the patient's incision had healed, what is the probability that it took more than 10 days to heal? Express the answer correct to three decimal places.

$\Pr(X > 10 | X < 13) = \frac{\Pr(10<X<13)}{\Pr(X<13)} = 0.563$

(1) use of conditional
(1) correct probability (3dp)

2 + 2 = 4 marks

b) Grainger selects the records of five discharged patients who have undergone this surgery. What is the probability that, for at least one patient, the incision had **not** healed at the time of discharge? Express the answer correct to three decimal places.

Let $Y$ =the number of patients with incision not healed at time of discharge.

$\boldsymbol{Y \sim Bi(5, 0.21186)}$
$\mathbf{Pr}(\boldsymbol{Y} \geq \mathbf{1}) = \mathbf{0.696}$

(1) binomial parameters
(1) correct probability (3dp)

2 marks

The hospital has recently purchased new technology for performing this surgical procedure. The manufacturer of the technology claims that the mean healing time will be reduced to 9 days and that for 95% of patients the incision will heal within 12 days.

c) If this claim is true, what is the standard deviation of the healing time when the surgery is performed using this technology? Express your answer in days, correct to two decimal places.

$$X \sim \mathrm{N}(9, \sigma^2)$$
$$\Pr(X < 12) = 0.95$$
$$\Pr\left(\frac{12-9}{\sigma}\right) = 0.95$$
$$1.64485 = \frac{12-9}{\sigma}$$
$$\sigma = 1.82\, days$$

(1) initial statement
(1) standardised value =1.64485
(1) correct standard deviation (2dp)

3 marks

After adopting this technology, the length of stay in hospital, T days, for patients undergoing this surgery is a continuous random variable with probability density function given by

$$f(t)=\begin{cases} k(t-8)^2(12-t) & 8\le t\le 12 \\ 0 & \text{otherwise} \end{cases}$$

d) Use calculus to show that $k=\dfrac{3}{64}$.

$$1=\int_8^{12} k(t-8)^2(12-t)\,dt$$

$$1=\left[\frac{-kt\left(3t^3-112t^2+1536t-9216\right)}{12}\right]_8^{12}$$

$$1=1728k-\frac{5120k}{3}$$

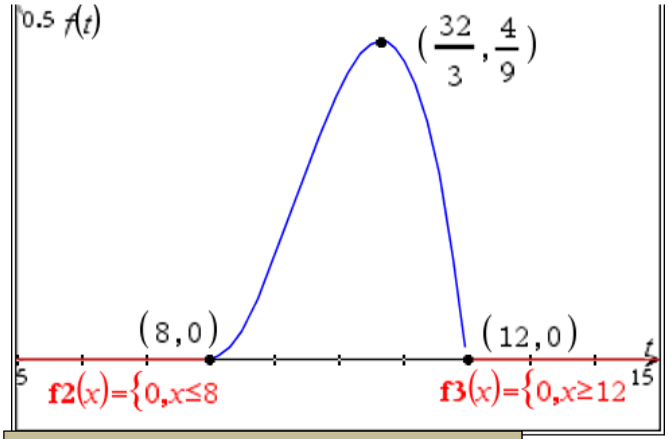
$$1=\frac{64k}{3}$$

$$k=\frac{3}{64}$$

(1) initial statement
(1) antiderivative
(1) substitution or equivalent values with subtraction

3 marks

e) Sketch the graph of $y=f(t)$.




(1) max point exact
(1) zero sections (1) shape + xints

3 marks

e) For these patients, what is the median length of stay in hospital, correct to two decimal places?

**Let $m$ be the median time in hospital.**

$$\int_8^m f(t)\,dt=\frac{1}{2}$$

$$t=10.46\ days$$

(1) correct use of integral
(1) correct time (2dp)

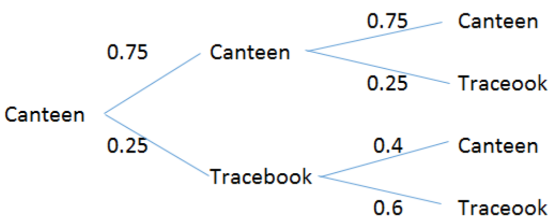
**NB: Do not accept rejections of other solutions based on the domain. The domain of this function is all real numbers. Students should reject solutions based on not being part of the interval [8,12]**

2 marks

During her lunch breaks, Grainger either goes to the canteen or she uses a ‘social networking’ website, Tracebook, to trace her childhood friends. If she uses Tracebook one day, there is a 60% chance that she will use Tracebook the following day. However, if she goes to the canteen one day, there is a 75% chance that she will go to the canteen the following day.

Grainger went to the canteen on Monday.

e) Use a suitable technique to demonstrate this situation for the first three days (Mon-Wed) of the week.




(1) start with Canteen and correct first branch
(1) correct second branches

2 marks

f) What is the probability that she will use Tracebook during the lunch break either on Tuesday or Wednesday, but not both?

$\mathbf{Pr}(\boldsymbol{T\ on\ one\ day}) = \mathbf{Pr}(\boldsymbol{CT}) + \mathbf{Pr}(\boldsymbol{TC})$

$\mathbf{Pr}(\boldsymbol{T\ on\ one\ day}) = \mathbf{0.75 \times 0.25 + 0.25 \times 0.4 = 0.2875}$

(1) Pr(CT)+Pr(TC)
(1) correct probability exact

2 marks

**Question 2 (11 marks)**

John keeps hens in his backyard. He regularly records the weights of the eggs that they lay and finds that the weights are normally distributed with a mean of 61 grams and a standard deviation of 8 grams. One afternoon John checks to find a fresh laid egg in the hen coop.

a) Calculate the probability, correct to four decimal places, that the egg weighs more than 67 grams.

$\mathbf{Pr}(\boldsymbol{X} > \mathbf{67}) = \mathbf{0.2266}$

1 mark

b) Calculate the probability, correct to four decimal places, that the egg weighs more than 67 grams, given that he knows it weighs more than 61 grams.

$\mathbf{Pr}(\boldsymbol{X} > \mathbf{67} | \boldsymbol{X} > \mathbf{61}) = \frac{\boldsymbol{Pr}(\boldsymbol{X}>\mathbf{67})}{\boldsymbol{Pr}(\boldsymbol{X}>\mathbf{61})} = \mathbf{0.4533}$

(1) use of conditional
(1) correct probability (4dp)

2 marks

The next morning, John finds 6 freshly laid eggs in the coop.

c) Find the probability that at least two of the eggs weigh more than 67 grams.

$\boldsymbol{Y \sim Bi}(\mathbf{6, 0.2266})$ where $Y$ represents the number of eggs that weigh more than 67 grams

$\mathbf{Pr}(\boldsymbol{Y} \geq \mathbf{2}) = \mathbf{0.4098}$

(1) binomial parameters
(1) correct probability

2 marks

John's neighbors, Kath and Kim, also keep hens, and they lay eggs whose weights are normally distributed with a standard deviation of only 2 grams. Kath and Kim brag that 98% of their eggs weigh more than 67 grams.

d) Find the mean weight of Kath and Kim's eggs. Give your answer correct to four decimal places.

$$\Pr(X > 67) = 0.98$$

$$\Pr\left(\frac{67-\mu}{2}\right) = 0.98$$

$$-2.05375 = \frac{67-\mu}{2}$$

$$\mu = 71.1075$$

(1) standardized value =-2.05375
(1) correct mean (4dp)

2 marks

John decides to set up a small business selling his eggs. He adopts the following price structure:

- 20 cents for an egg that weighs less than 53 grams
- 40 cents for an egg that weighs more than 77 grams
- 30 cents for all other eggs

e) Set up a probability distribution table for the profit for John's small business.

| $p$ | 0.2 | 0.4 | 0.3 |
|---|---|---|---|
| $\Pr(P = p)$ | 0.15866 | 0.02275 | 0.81860 |

(1) one correct probability
(1) all correct in table

Note students may use cents.

2 marks

f) Find the expected profit for 100 eggs.

$$\boldsymbol{E(P) = 0.2 \times 0.15866 + 0.4 \times 0.02275 + 0. \times 0.818}$$

$$\boldsymbol{E(P) = 0.2864}$$
$$\boldsymbol{E(100P) = 100 \times 0.2864 = \$28.64}$$

(1) suitable calculation
(1) $28.64

2 marks