2018 SAC 3 PREP 5 - SOLUTIONS

Question 1

The probability that seeds from a certain species of plant are non-viable is $\frac{1}{300}$.

a. How many viable seeds would you expect in a batch of one million seeds?

$$X \sim Bi\left(1000000, \frac{299}{300}\right)$$

$$\mu = E(X) = \frac{2990000}{3}$$

2 marks

b. State a 95% confidence interval for the number of viable seeds in a packet of 100 seeds.

$$\mu = E(X) = 100 \times \frac{299}{300} = \frac{299}{3}$$

$$\sigma = \sqrt{100 \times \frac{299}{300} \times \frac{1}{300}} = \frac{\sqrt{299}}{30}$$

$$\left(\frac{299}{3} - 2 \times \frac{\sqrt{299}}{30} \le X \le \frac{299}{3} + 2 \times \frac{\sqrt{299}}{30}\right)$$

$$98.51 < X < 100.82$$

$$X \in [99,100]$$

2 marks

c. A packet contains 100 seeds chosen at random. Find the probability that there are less than 99 viable seeds, correct to 4 decimal places.

$X \sim Bi\left(100, \frac{299}{300}\right)$ $\qquad$ $\Pr(X < 99) = \Pr(X \le 98) = 0.0444$

1 mark

d. If a packet contains less than 99 viable seeds, the customer receives a full refund on their purchase.

i) If the profit on a packet of seeds is \$1.20 and a refund results in a loss of 50 cents, calculate the expected profit from a packet of seeds. (to the nearest cent)

\$x	Profit \$1.20	Loss \$0.50
Pr(X=x)	0.9556	0.0444

E(P) = (\$1.20 x 0.9556) - (\$0.50 x 0.0444) = \$1.12

2 marks

ii) If the seeds are redistributed to sellers in boxes of 25 packets, find the probability that 5 out of each box of 25 packets will result in a refund. (correct to 4 decimal places)

Binomial distribution: $X \sim Bi(25, 0.044358)$

Let X = Number of packets

$\Pr(X = 5) = 0.0037$

2 marks

The probability that viable seeds develop into mature plants is a function of the amount of light that the plant is exposed to, the quality of the soil as well as the level of nutrients in the soil. Of particular importance is the concentration of soluble nitrates in the soil.

The probability of a viable seed developing into a mature plant is modelled by the function

$$f(x) = \begin{cases} k - k(x-1), & \text{for Domain A} \\ kx, & \text{for Domain B} \\ 0, & \text{elsewhere} \end{cases}$$

Where x represents the concentration of soluble nitrates in g/L for $0 \le x \le 2$ and k is a positive integer value.

e. **i.** State the interval corresponding to Domain A for the model.

$x - 1 \ge 0$

But as $0 \le x \le 2$, then:

$1 \le x \le 2$

1 mark

ii. State the interval corresponding to Domain B for the model.

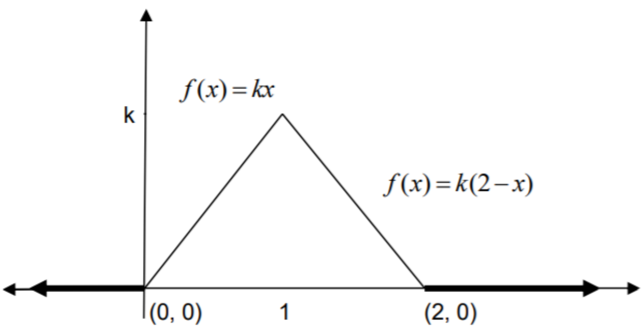
$x - 1 < 0$

But as $0 \le x \le 2$, then:

$0 \le x < 1$

1 mark

iii. Without solving for k sketch the graph of f labelling all key features.



2 marks

f. Find the value of k such that f is a probability density function.

For PDF Area = 1

Symmetric graph so Area of triangle = 0.5

Area = $\frac{1}{2} \times 1 \times k = \frac{1}{2}$

$\therefore k = 1$

2 marks

g. Find the probability that a viable seed will develop into a mature plant in soils containing concentrations of greater than 0.5 g/L of soluble nitrates.

Need $\Pr\left(X > \frac{1}{2}\right)$

$$\Pr\left(X > \frac{1}{2}\right) = \int_{\frac{1}{2}}^{1} x\,dx + \int_{1}^{2} 2 - x\,dx = \frac{7}{8}$$

2 marks

h. Find the standard deviation of *X* and hence provide a 95% confidence interval for the distribution of *X*.

$$Var(X) = \int_{0}^{1} x^2 \times x\,dx + \int_{1}^{2} x^2(2-x)\,dx - \mu^2$$

Gives Var(X) $= \frac{1}{6}$

Gives sd(X) $= \frac{1}{\sqrt{6}}$

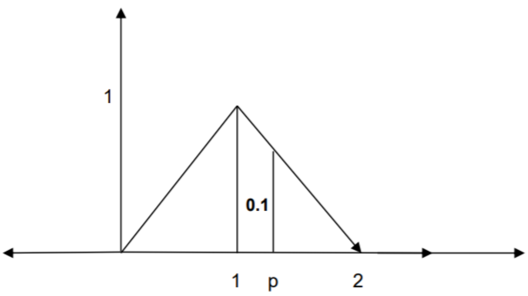
95% confidence interval:

$$\therefore \mu - 2\sigma \le X \le \mu - 2\sigma = 1 - \frac{2}{\sqrt{6}} \le X \le 1 + \frac{2}{\sqrt{6}}$$

2 marks

i. It is known that the probability of a viable seed developing into a mature plant is 0.6 provided that the concentration of soluble nitrates is *p* g/L. Find the value of *p*.

$\Pr(X < p) = 0.6$



From the graph: $\Pr(1 < X < p) = 0.1$

Need: $\int_{1}^{p} (2 - x)\,dx = 0.1$

$$p = \frac{2\left(\sqrt{5} \pm 5\right)}{5}$$

Within domain gives $p = \frac{10 - 2\sqrt{5}}{5}$.

3 marks

Total 22 marks