Name: _____

Teacher: _____

School No: _____

Scotch ✠ College

Department of Mathematics

2017

Mathematical Methods (CAS)

Unit 3 SAC 1b
Application Task

No calculator allowed

No notes allowed

Date: Monday 5th June (Day 2)

Time: 45 minutes

Marks: 34

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this task are **not** drawn to scale.

1   The graph of the function

$$f : [-\frac{1}{2}, \frac{5}{2}] \to \mathbb{R}, f(x) = ax^2(b-x)^2 \text{ , where } \{a,b\} \in \mathbb{R} \text{, is shown.}$$
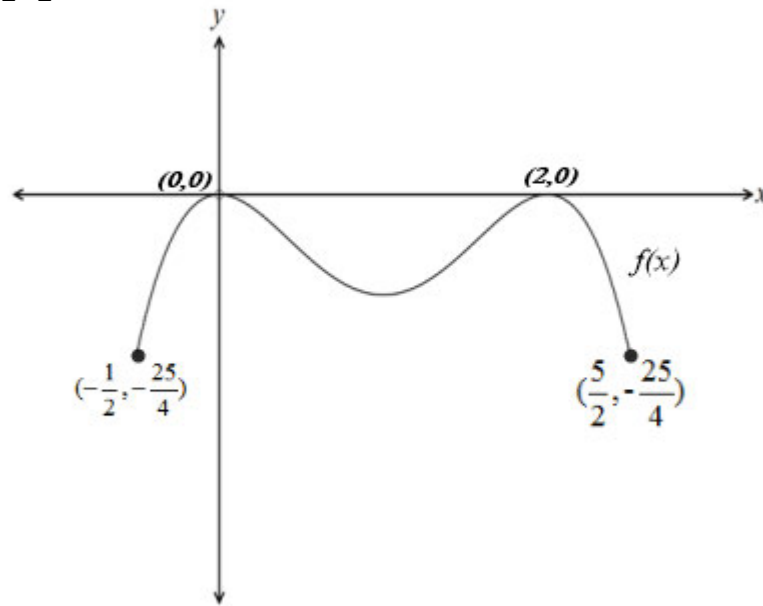
a   from the information provided on the graph above, find the values of a and b.

b   **i**   Find $f'(x)$ in terms of a and b and express your answer as a product of three linear factors.

ii   Use your values of a and b from part **1a** to evaluate $f'\left(\dfrac{1}{2}\right)$.

c   State the coordinates of the minimum stationary point.

d   **i**   Let $f(x) = w$, where $w \in \mathbb{R}$.

State the set of values of w for which the graph with equation $f(x) = w$ has **two** real solutions.

ii A transformation is described through $X' = TX + B$ where

$T = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Find the equation of the image of f

using your values of a and b in **1(a)** and sketch the graph of the image on the set of axes provided below. Show clearly the coordinates of the turning points only (coordinates of the intercepts are not required).
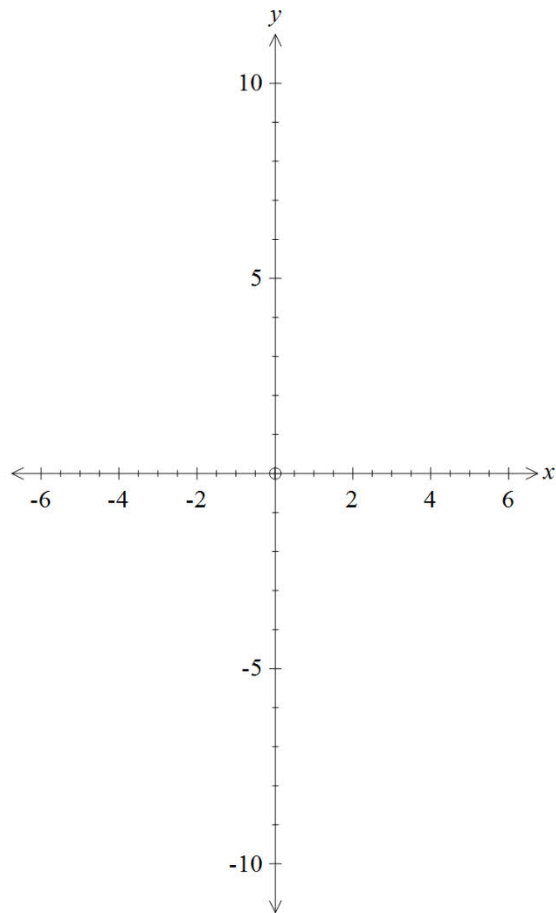

The graph shows turning points at $(0,0)$, $(2,0)$, $\left(-\frac{1}{2}, \frac{25}{4}\right)$ and $\left(\frac{5}{2}, -\frac{25}{4}\right)$.
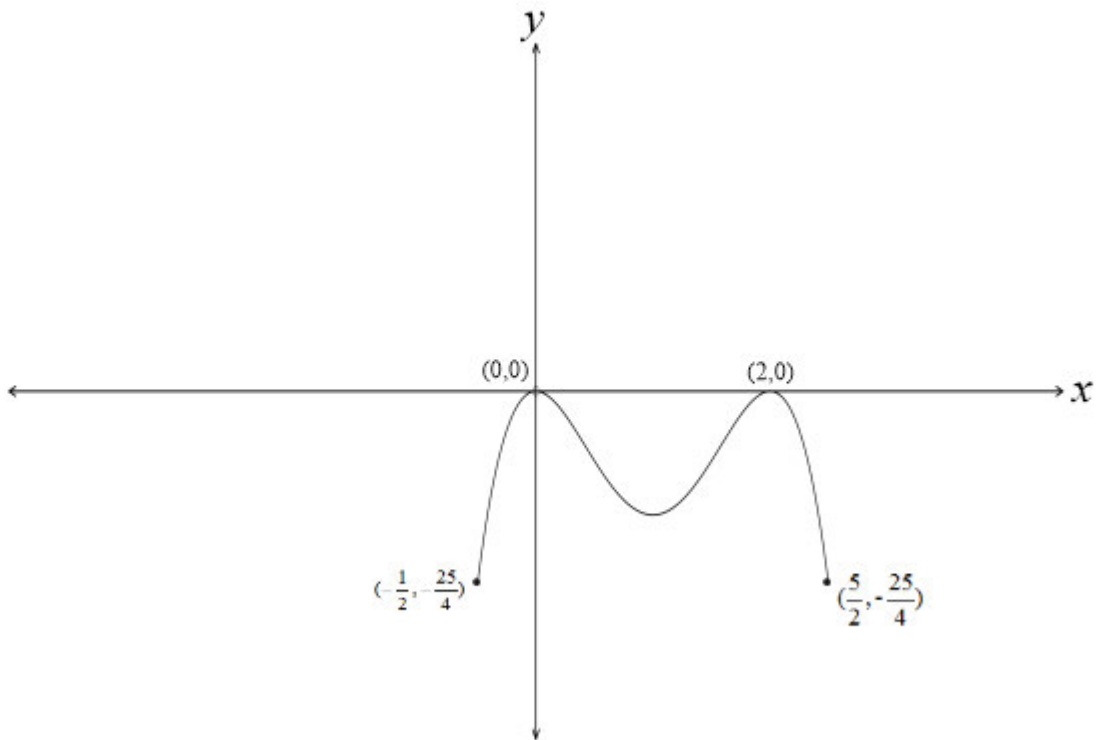
(Marks: 2 + (2 + 1) + 1 + (1 + 4) = 11)

2   **a**   Consider the functions:

$$h:\mathbb{R}\to\mathbb{R}, h(x)=x^{\frac{2}{3}} \text{ and } g:[0,\infty)\to\mathbb{R}, g(x)=x^{\frac{5}{2}}$$

     i   Fully define the composite function $c(x)=h(g(x))$

     ii   Sketch the graph of c showing axis intercepts.

$$h:\mathbb{R}\to\mathbb{R}, h(x)=x^{\frac{2}{3}} \text{ and } g:[0,\infty)\to\mathbb{R}, g(x)=x^{\frac{5}{2}}$$
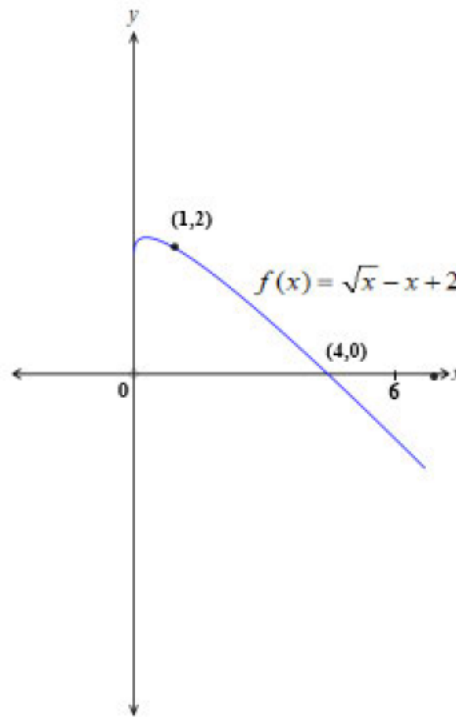
b   The graph of $y = c(x)$ is dilated by a factor of 2 from the x axis and translated 1 unit in the negative direction of the x-axis.

The equation of the image is $y = a(x+b)^{\frac{c}{d}}$. Find the values of a, b, c and d.

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(Marks: $(2 + 2) + 2 = 6$)

3   The graph of the function $f : [0, 6] \to \mathbb{R}, f(x) = \sqrt{x} - x + 2$ is shown below.

The equation of the normal at the point where $x = 1$ is $y = 2x$

a   Find the equations of both the tangent and the normal at the coordinate point where $x = 4$.

b   The lines found in part **a** are drawn in the following diagram. Given that point

B is $\left(\dfrac{12}{11}, \dfrac{24}{11}\right)$, find the coordinates of point A and hence the area of the

shaded region in $\triangle ABC$.
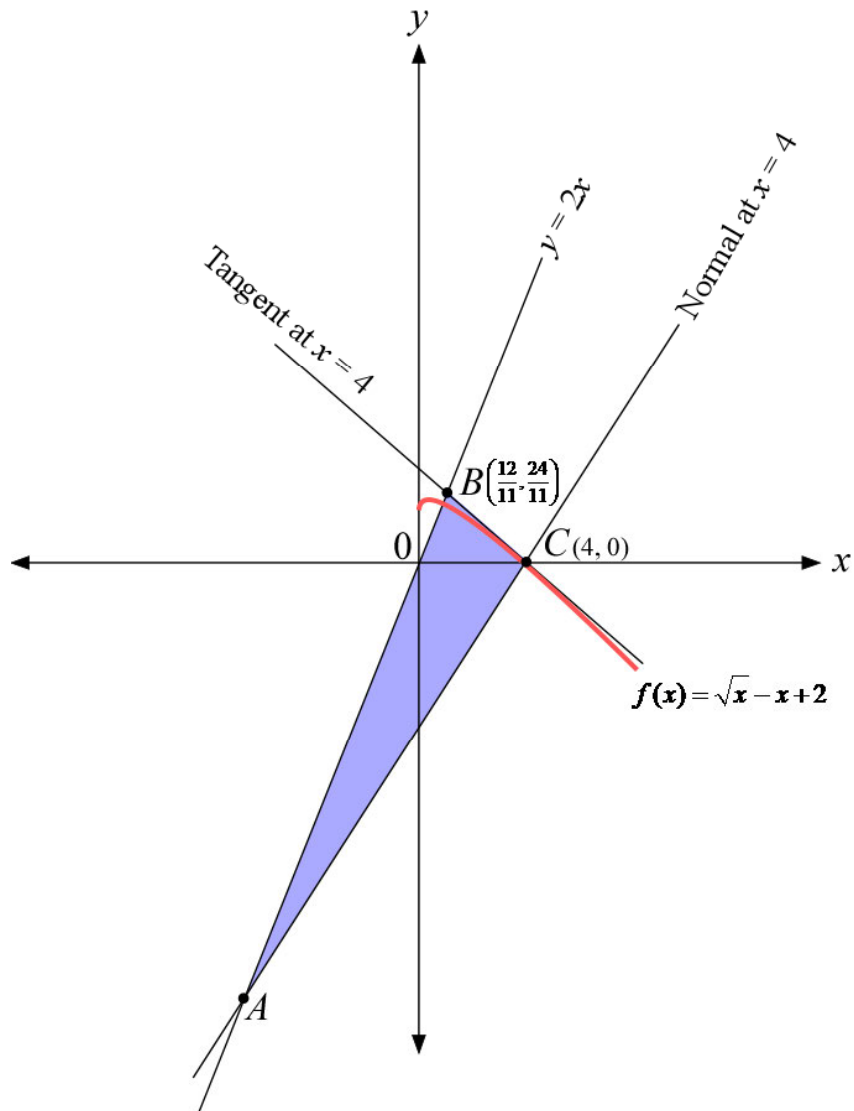
(Marks: 2 + 3 + 3 = 8)

4   **a**   State the maximal domain of $y = \sqrt{-x^2 + 4x - 3} + 1$

b   A girl has a circular leadlight window that she enjoys watching most of the time. The only times when she is not happy with the view to the window is when the sun reflects the shadow of the electricity line on the window, as shown below.
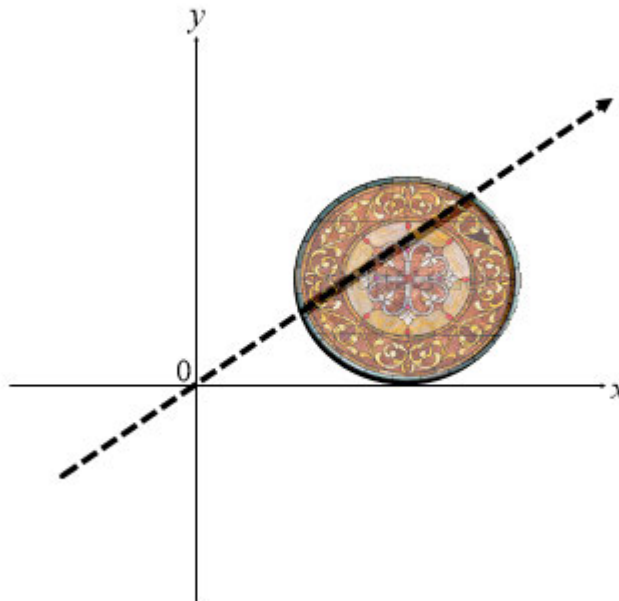
The equation of the circular window is $(x - 2)^2 + (y - 1)^2 = 1$ and the shadow of the electricity line is a section of the line $y = mx$.

i   Find the value(s) of *m* when the shadow is not shown in any part of the window.

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ii   Find the coordinates of the points where the shadow could be a tangent to the circumference of the circular window.

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(Marks: $2 + 4 + 3 = 9$)

End of SAC 1b