Name: ______________________________

Teacher: ____________________________

School No: _____________

Scotch College

Department of Mathematics

2017

Mathematical Methods

Unit 4 SAC 1a

Modelling Task- Project Component

Students are advised NOT to use their CAS calculator

Distribution Date:	Wednesday 26th July (Day 2)
Due Date:	Thursday 3rd August (Day 2)
Total Marks:	40

Instructions

Answer **all** questions in the spaces provided.

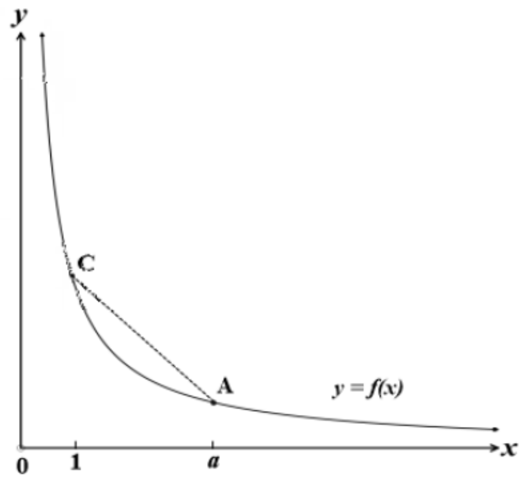
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this task are **not** drawn to scale.

Question 1

The diagram below shows part of the graph of the function $f: \mathbb{R}^+ \to \mathbb{R}, f(x) = \frac{5}{x}$



The line segment CA is drawn from the point $C(1, f(1))$ to the point $A(a, f(a))$ where $a > 1$.

a **i** Calculate the gradient of CA in simplest form, in terms of a.

ii At what value of x between 1 and a does the tangent of the graph of f have the same gradient as CA?

b **i** Calculate $\int_a^e f(x)\,dx$ in terms of a.

ii Let b be a positive number less than one. Find the exact value of b such that $\int_b^1 f(x)\,dx = 5$.

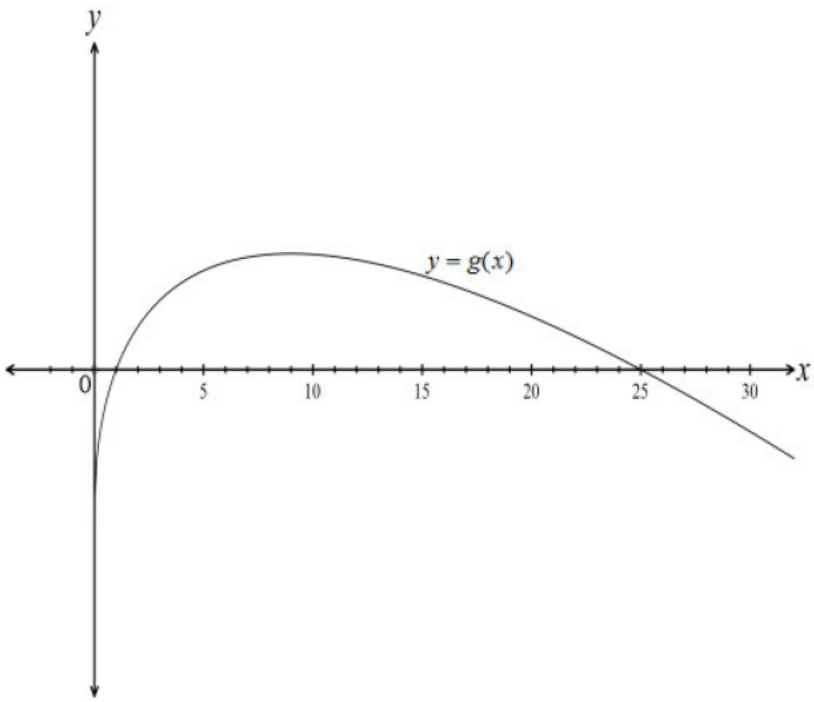
c **i** Express the area of the region bounded by the line segment CA, the x-axis, the line $x = 1$ and the line $x = a$ in terms of a and in simplest form.

ii For what exact value of a does this area equal 5?

((2 + 2) + (2 + 2) + (2 + 3) = 13 marks)

Question 2

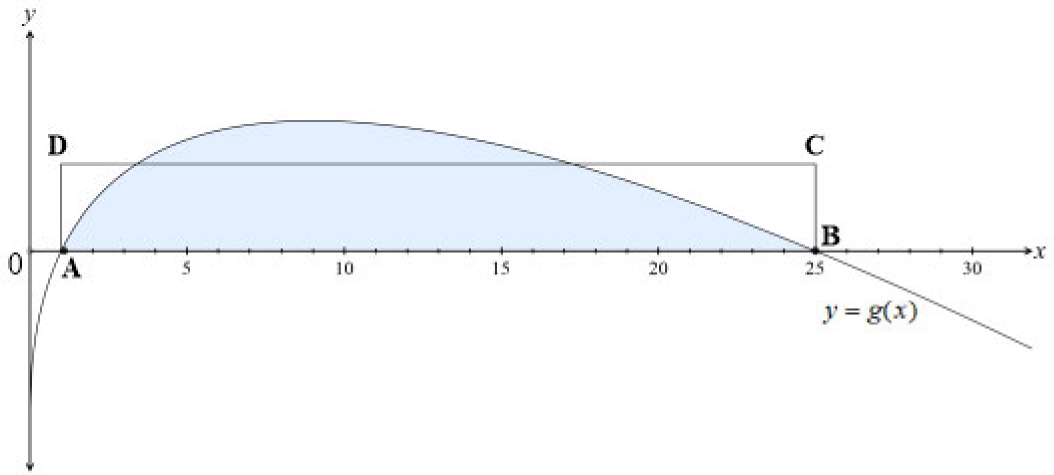
Let $g : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}, \quad g(x) = 6\sqrt{x} - x - 5$. The graph of $y = g(x)$ is shown below.



a State the interval for which the graph of g is increasing.

b Points A and B are the points of intersection of $y = g(x)$ with the x-axis. Point A has coordinates (1 , 0) and point B has coordinates (25 , 0). The area of the shaded region is 64 squared units.

Find the length of AD such that the area of the rectangle ABCD is equal to the area of the shaded region.



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c By finding the average value of function g from $x = 1$ to $x = 25$, and using calculus, verify your answer to part **b**.

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d Given that $\int_{1}^{25} g(x)\,dx = 64$, find:

i $\int_{1}^{25} \left(\frac{g(x)}{2}\right) dx$

ii $\int_{1}^{25} 2g(x)\,dx - 72$

iii $\int_{1}^{25} 2\left[g(x) - 20\right] dx$

iv $\int_{25}^{1} \left(\frac{g(x)}{4}\right) dx$

v $\int_{0\ 5}^{12\ 5} g(2x)\,dx$

(1 + 1 + 2 + (1 + 1 + 1 + 1 + 1) = 9 marks)

Question 3

a Consider the function $h:[0,2] \to \mathbb{R}$, $h(x) = 2\sin(2\pi x - \frac{\pi}{2}) - 1$

i State the amplitude of h

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ii State the period of h

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iii Find the general solution for the values of x if $h(x) = 0$.

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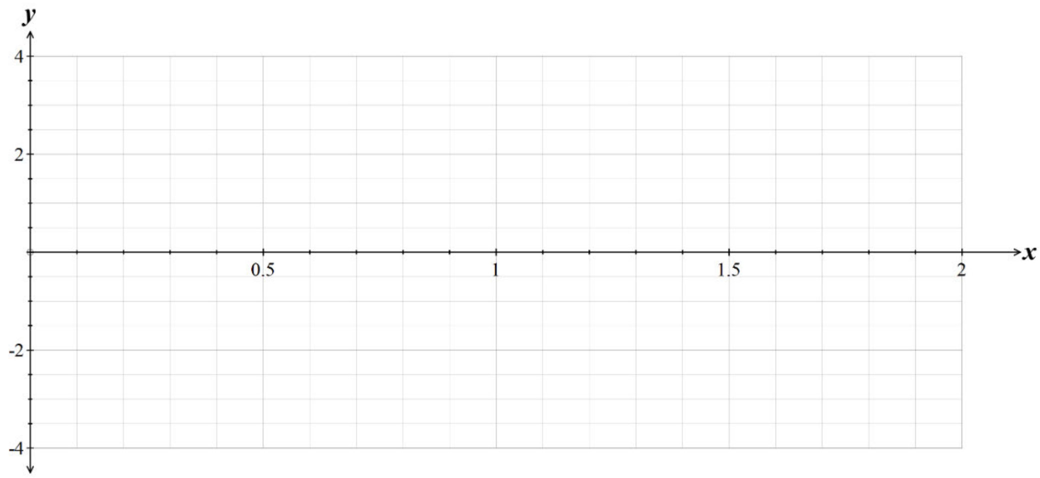
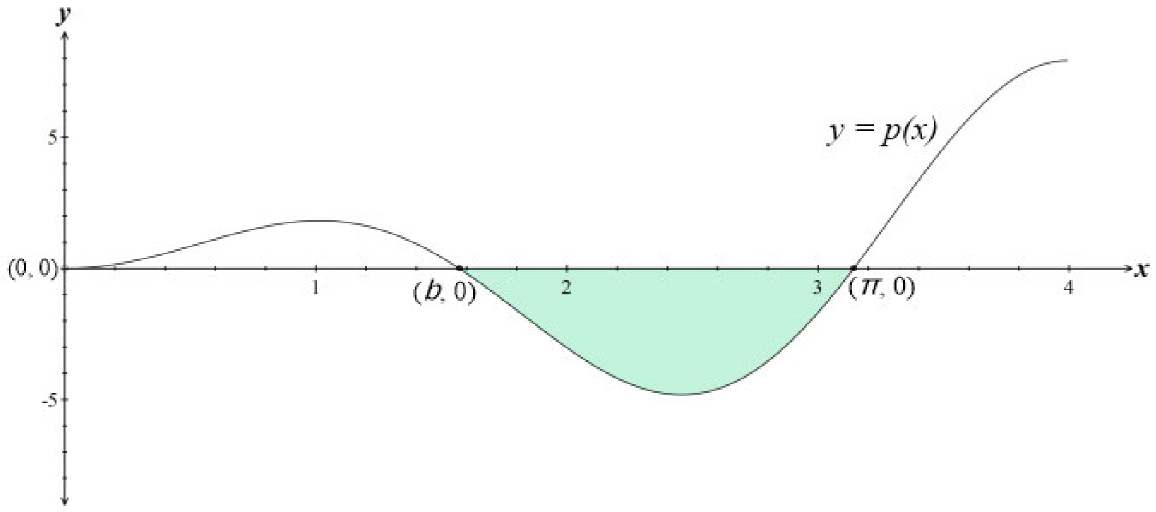
iv Use the above information and the set of axes below to sketch the graph of $y = h(x)$. Show clearly all the coordinates of the x-intercepts and the end points.



v On the same set of axes above sketch the graphs of $y = 3x - 2$ and $x = \frac{3}{2}$

vi Find the area of the region bounded by $y = h(x)$, the line $y = 3x - 2$ and the line $x = \frac{3}{2}$, expressing your answer in the form of $\frac{a\pi + b\sqrt{3}}{c\pi}$.

b Let function p be defined as $p:[0,4]\to\mathbb{R},\ p(x)=2x\sin(2x)$. The graph of p is shown below where $y=p(x)$.



i Find the value of *b*, one of the *x*-intercepts of $y=p(x)$, as indicated on the graph above.

ii Find the derivative of $y=x\cos(2x)$.

iii Hence find the area of the shaded region enclosed by the curve $y = p(x)$ between $x = b$ and $x = \pi$ using your value of b from **bi**.

((1 + 1 + 2 + 3 + 3 + 3) + (1 + 2 + 2) = 18 marks)

End of SAC 1a