Name: ______________________________

Teacher: ____________________________

School No: __________________________

Scotch College

Department of Mathematics

2017

Mathematical Methods

Unit 4 SAC 2a

Modelling Task

CAS calculator allowed

Distribution Date:	Thursday 31st August (Day 3)
Due Date:	Friday 15th September (Day 2)
Total Marks:	80

Instructions

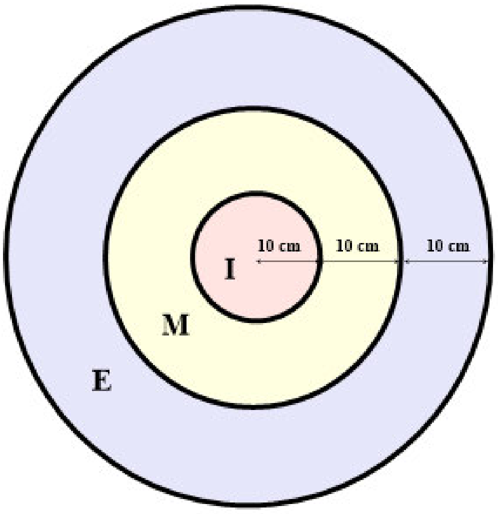
Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this task are **not** drawn to scale.

1 Jenny is throwing darts at the circular target of radius 30 cm. The target is divided by circles of radius 10 cm and 20 cm as shown in the diagram:



She scores 3 points for hitting the inner region (I), 2 points for hitting the middle region (M) and 1 point for hitting the edge region (E). Assume that when Jenny throws a dart, although she always hits the target, she is equally likely to hit any point on the target. This means that the probability of her hitting a region is proportional to the area of the region. (Assume that the outer circles of each region of the dart board are negligible.)

a Show that the areas of the regions E, M and I are in ratio 5:3:1.

b When Jenny throws once at the target, find the probability that she hits the region E of the target.

c Let X denote the score obtained by Jenny on one throw. Complete the following table for the probability distribution of X.

k	1	2	3
Pr(X=k)			$\frac{1}{9}$

d Find Jenny's expected score with one throw.

e Jenny throws two darts at the target. The two throws are independent.

i Find the probability that she scores a 2 both times.

ii Find the probability that the sum of two scores is 4.

f If Jenny were to throw at the target 20 times, about how many times would she score a 3?

g **i** Find an expression for the probability that she scores 3 at least once in n throws.

ii For what values of n is this probability greater than 0.99?

(Marks: 2 + 1 + 2 + 1 + (1 + 2) + 1 + (2 + 2) = 14)

2 The annual rainfall of Shtam is known to be an approximately normally distributed random variable with a mean of 330 mm and a standard deviation of 65 mm. A year is regarded as ‘very wet’ for Shtam if the rainfall exceeds 520 mm. The following table gives descriptions of other rainfalls.

Rainfall for the year	Description of the year
more than 520 mm	very wet
between 300 mm and 520 mm	moderate
between 220 mm and 300 mm	fairly dry
between 180 mm and 220 mm	very dry
less than 180 mm	drought year

a State the probability that the rainfall in Shtam in 2018 will exceed 330 mm.

b Calculate the probability that Shtam will have a rainfall that is ‘very wet’ for Shtam in 2018. Give your answer correct to four decimal places.

c Calculate the probability that a particular year will have a rainfall that is ‘fairly dry’ for Shtam. Give your answer correct to four decimal places.

d Assuming that the weather for any year is independent of the weather for any other year, find:

i the probability that in a given three-year period, all three years will be 'fairly dry' for Shtam. Give your answer to four decimal places.

ii the probability that in a given seven-year period, exactly three years will be 'fairly dry' for Shtam. Give your answer correct to four decimal places.

e How many millimetres of rainfall annually are expected about 95 per cent of the time? Give your answer correct to the nearest millimetre.

(Marks: 1 + 1 + 1 + (1 + 1) + 2 = 7)

3 Karen is a contestant in the shot-put event at the world championships. In a particular throw, Karen throws X metres. X is a normally distributed random variable with mean 19.2 and variance 0.36.

a For any throw, find to four decimal places the probability that Karen throws

i more than 19.6 metres.

ii less than 17.8 metres.

iii between 17.8 and 19.6 metres.

During the championships each competitor in the shot put event has five throws.

b In Karen’s five throws, find the probability, correct to three significant figures, that:

i Karen’s first three throws are all less than 19.6 metres and both her last two throws are more than 19.6 metres.

ii At least three of Karen’s throws are more than 19.6 metres.

iii Karen’s final throw is more than 19.6 metres, given that at least three of her five throws are more than 19.6 metres.

c Between what two distances, symmetrically placed about the mean, would 95 percent of Karen's throws be expected to lie? Give your answers to two decimal places.

d During training for the championships, Karen had sixty practice throws. How many of these throws would be expected to be more than 19.6 metres?

After her five world championship throws, Karen's best throw was 21.1 metres. She was leading the event when a competitor, Kelly, still had a final throw. In a particular throw, Kelly throws Y metres; where Y is a normally distributed random variable with variance 0.81.

e If the probability that Kelly's final throw is more than 21.1 metres is 0.26, find the mean of the distances that Kelly throws, correct to two decimal places.

(Marks: (1 + 1 + 1) + (1 +1 + 2) + 2 + 2 + 3 = 14)

4 Hugo has a spinner subdivided into 6 regions. Each region is represented by the numbers one to six.

When the spinner is spun, the probability of landing on a particular region, X, is shown in the following table.

X	1	2	3	4	5	6
$\Pr(X = x)$	$2p$	$2p$	$3p$	$3p$	$5p$	q

a if $p = \frac{1}{20}$ find the exact value of q.

b Hugo spun the spinner twice.

i Find the exact probability that both the scores on the spinner were sixes.

ii Find the exact probability that the sum of the scores was 9.

iii Find the exact probability that, if the sum of the scores was 9, then one of the scores was 6.

c Hugo spun the spinner 5 times. What, correct to four decimal places, is the probability of landing **at least twice** on two?

d Hugo spun the spinner n times. Find the least value of n for which the probability of landing **at least once** on 2 is greater than 0.5.

(Marks: 2 + (1 + 2 + 2) + 1 + 3 = 11)

5 A food distributor sells remnants of meat in bags. The weight, in kg, of remnants is a continuous random variable, X, with probability density function.

$$f(x) = \begin{cases} k(x-1)(3-x), & 1 < x < 3 \\ 0 & x \le 1 \, and \; x \ge 3 \end{cases}$$

a Show that $k = \frac{3}{4}$

b Find the probability that one of these bags has weight greater than 2.5 kg. Express your answer as a fraction.

c What is the median weight of these remnant bags?

At the end of a particular day, the wholesaler has 20 bags of meat remnants. He labels the bags as small, medium and large on the basis of their weight. A bag is labelled small if it has at most 1.5 kg of remnants in it.

d Find the probability he has at least one small bag. Give your answer correct to four decimal places.

A meat wholesaler claims that he sells meat in bags of 5 kg each. The weights of these bags are approximately normally distributed. The mean weight of these bags is 5.1 kg and it is known that 10% of these bags have less than 5 kg meat in them.

e **i** Find the standard deviation of the weights of these bags correct to four decimal places.

ii Find the percentage of bags that weigh at least 5.3 kg. Give your answer correct to four decimal places.

f The meat wholesaler developed a function that represents the distribution of the weight of the entire production of these 5kg bags of meat remnants.

$$f(x)=\frac{1}{0.08\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-5.1}{0.08}\right)^2}$$

Use the information of the above function to find $E\left(X^2\right)$ to the nearest whole number.

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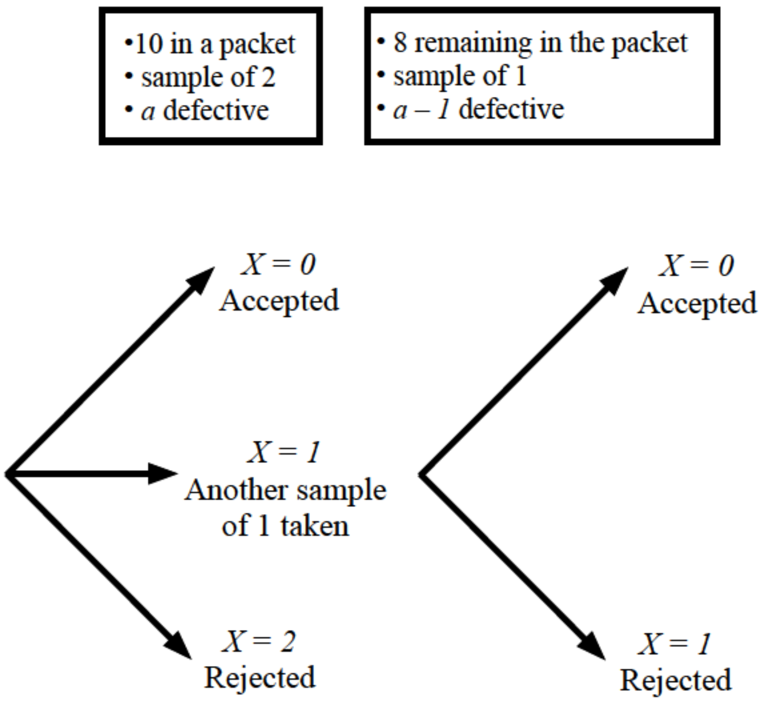
(Marks: 2 + 2 + 2 + 3 + (3 + 1) + 2 = 15)

6 **a** A company is producing a new compact type of fuel injector for Maserati cars. They are sold to the manufacturers of these cars and different car service departments around the world. These fuel injectors are sold in packets of 10. From long term experience the fuel injectors makers know that there are *a* defective injectors in every packet.

Inspectors from major buyers test the product by sampling randomly 2 injectors from a packet of 10. Let X be the number of defective injectors in the sample of 2 selected. The decision of making a purchasing order or not relies on the following conditions as described in the diagram below.

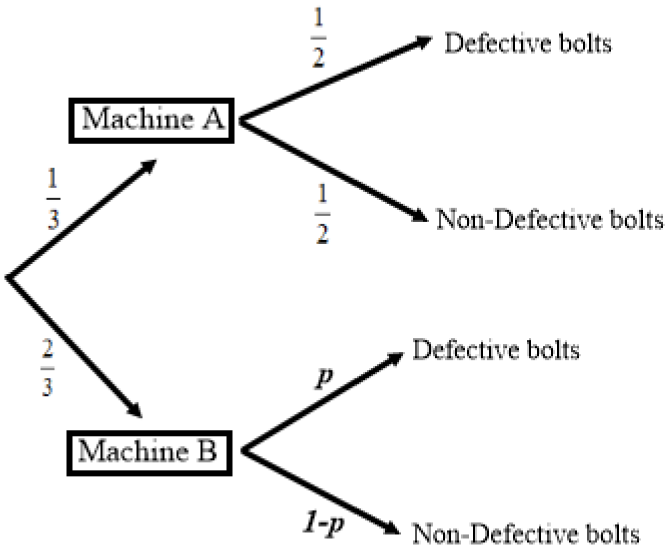


i Show that the probability of accepting the sample and therefore making an order in terms of a is $\frac{(10-a)(9-a)(4+a)}{360}$.

ii If the sales department is aiming for a probability of acceptance to be $\frac{2}{3}$, find the value of a that may achieve this result.

iii On a particular day the inspector sampled two packets, each containing 10 injectors. Find the value of a if the probability of both packets being accepted was $\frac{2401}{3600}$.

b In a different production plant, there are two machines producing a particular type of a bolt that is commonly used in car engines. Unfortunately, both machines can produce defective bolts. The defective bolts are produced independently of any previous defective bolts. Over a long period of time, the plant manager observed the daily production and constructed a tree diagram of the proportions of defective and non-defective bolts produced by the two machines as shown below.



If a bolt was randomly selected from the day's production;

i find the probability in terms of ***p*** that the bolt selected was defective.

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ii If a selected bolt was defective, find the probability, in terms of p, that it came from Machine B. Express your answer in the form $\frac{ap}{ap+b}$ where a and b are positive integers.

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iii For **iii , iv** and **v** consider $p = 0.25$

Show that the probability that a randomly selected bolt from the day's production is non-defective is $\frac{2}{3}$.

iv Find the exact probability that from five randomly selected bolts from the day's production, the first two are defective and the remaining three are non-defective.

v find how many bolts should be selected in order to obtain a probability greater than 90% of selecting at least two defective bolts.

vi Find in simplest form and in terms of *p*, (*p* is no longer 0.25) the probability that from a randomly selected sample of 5 bolts, exactly two are defective.

vii Find the value of p which will regulate machine B to produce the maximum probability of selecting exactly two defective bolts in every 5 randomly selected bolts from the day's production.

(Marks: (2 + 1 + 2) + (2 + 2 + 1 + 1 + 3 + 2 + 3) = 19)

END OF SAC.