Name: _____

Teacher: _____

School No: _____

Scotch College

Department of Mathematics

2017

Mathematical Methods

Unit 4 SAC 2b

Test - Modelling Task

CAS calculator allowed

A single bound book or log-book is allowed

Formula sheet provided

Date: Friday 15th September (Day 2)

Total Marks: **40**

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this task are **not** drawn to scale.

1 Woody, the champion soccer player, buys all his mandarins in bags from one of Shtam's popular supermarkets. At this supermarket the number of mandarins, X, in a **large** bag is a discrete random variable with probability distribution as shown in the following table:

x	24	25	26	27
$\Pr(X = x)$	$\frac{1}{4k}$	$\frac{k}{12}$	$\frac{k+2}{k^2+3}$	$\frac{k}{2k+6}$

a Find all possible values of k. Give your answer to three decimal places where appropriate.

(Marks: 2)

At the supermarket the number of mandarins, Y, in a **small** bag is a discrete random variable with probability distribution as shown in the following table:

y	14	15	16	17
$\Pr(Y = y)$	0.21	0.38	0.23	0.18

b Woody buys two small bags of mandarins.

Find the probability, correct to four decimal places, that

i each bag has 15 mandarins.

ii the total number of mandarins in both small bags is equal to 31.

iii if it is known that the total number of mandarins in both bags is equal to 31 then one of the bags has 15 mandarins.

(Marks: 1 + 2 + 1 = 4)

c On another occasion Woody buys seven small bags of mandarins.

Find the probability, correct to four decimal places, that

i at least three of the bags have 14 mandarins.

ii if at least two of the bags have 15 mandarins then exactly four of the bags have 15 mandarins.

(Marks: 1 + 2 = 3)

Woolley, Woody's friend also buys his mandarins from the same supermarket. He buys a number of small bags of mandarins to take with him to the next soccer game.

d **i** Find the smallest number of bags that Woolley should buy if the probability of **at least one** of his bags containing **at least 16** mandarins is greater than 0.86.

A regular mandarin has a width of at least 7 cm. One of the small bags that Woolley buys has 15 mandarins in it and 12 of those are regular.

ii Woolley eats 3 mandarins selected from this bag. Find the probability that less than two of the mandarins in the sample are not-regular.

(Marks: 2 + 2 = 4)

e Woolley noticed that Woody is not consistent with his soccer training attendance. Over a period of time he noticed that Woody attends training once every five training sessions. Woolley also noticed that he takes mandarins for the boys at soccer games twice in every three games.

Let event A represent Woody's attendance in training and event B be Woolley's frequency of taking mandarins to training session, where $\Pr(A)=\frac{1}{5}$ and $\Pr(B)=\frac{2}{3}$.

If A' denotes the complement of A, calculate $\Pr(A' \mid B)$ when:

i A and B are independent.

ii A and B are mutually exclusive.

iii $\Pr(A \cup B) = \frac{3}{4}$.

(Marks: 1 + 1 + 3 = 5)

The weights of the bags of mandarins follow a normal distribution with mean 1.5 kilograms and standard deviation of 0.4 kilograms. Let X represent the weight of the bags in kilograms.

f **i** Find the probability of a randomly selected bag of mandarins weighing less than 1.5kg.

ii Find the probability, to four decimal places that of a randomly selected bag of mandarins weighing at least 1.65 kg.

iii Given that a bag of mandarins selected at random was weighing between 1.5 kg and 1.7 kg, find, to four decimal places, the probability that it was at least 1.65 kg.

iv The heaviest 10% of the bags have the best mandarins. Find, to two decimal places, the weight of the lightest possible bag of the best mandarins.

v Find the exact value of a such that $\Pr(X < a) = \Pr\left(Z > \frac{a}{3}\right)$, where Z is the standard normal random variable, such that $Z \sim N(0,1)$.

vi Find x_1 and x_2 if the $\Pr(x_1 \leq X \leq x_2) = 0.7$. Where x_1 and x_2 are symmetrical around the mean. Give your answer to two decimal places.

(Marks: 1 + 1 + 2 + 1 + 3 + 2 = 10)

g Woody found that on a particular season the packaging of the bags relative to the weight of mandarins followed the normal distribution where $f(x)=\frac{2}{\sqrt{2\pi}}e^{-\frac{1}{2}(2(x-5))^2}$.

Find $E(X^2)$.

(Marks: 2)

h On another season Woolley found that 5% of the bags packaged were greater than 1.7 kg and 20% of bags were less than 1.4 kg. Find, to two decimal places, the mean and the standard deviation of the normal distribution observed by Woolley.

(Marks: 3)

i The weather in Shtam in 2017 has been unusual and has effected the production of mandarins. Woody suspects that the packaging of mandarins for the next season will not be normally distributed. In fact he calculated that the bags of mandarins will be a continuous density distribution f as follows, were x is in kilograms.

$$f(x)=\begin{cases} kx^2(x-2) & \text{if } 1<x\leq 2 \\ \\ 0 & \text{otherwise} \end{cases}$$

i Show using Calculus that the value of k so that the function is a probability density function is equal to $-\frac{12}{11}$.

ii Find, to two decimal places, the mean and the median of Woody's suggested density function.

iii Find, to two decimal places, the variance of Woody's suggested distribution.

iv Find, to four decimal places, the $\Pr(\mu - 2\sigma < x < \mu + 2\sigma)$ of Woody's distribution.

(Marks: 2 + 2 + 1 + 2 = 7)

End of SAC 2b.