Name: ______________________________

Teacher: ____________________________

School No: _____________

Scotch College

Department of Mathematics

2018

Mathematical Methods

Unit 3 SAC 1b

Application Task

No calculator allowed

No notes allowed

Date: Tuesday 5 June (Day 3)

Time: 45 minutes

Marks: 32

Instructions

Answer **all** questions in the spaces provided.

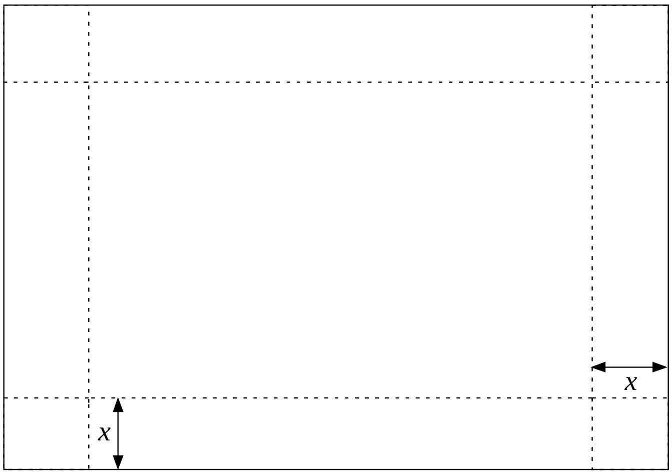
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this task are **not** drawn to scale.

There is no need to justify maxima or minima unless stated otherwise in this task.

1 An open rectangular box is to be constructed from a rectangular sheet of card.



If the length of the rectangular sheet of card is 30cm and the width is 15cm and squares of length $x$cm are cut from the corners to enable the box to be constructed,

a Find the rule for $A(x)$, the external surface area of the open box, in simplest form.

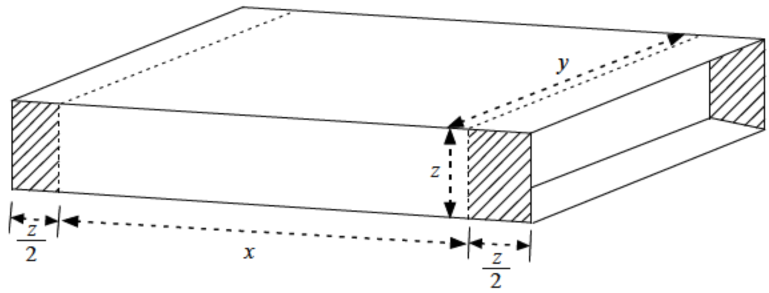
b State a suitable domain for x.

(Marks: 2 + 2 = 4)

2 If the sum of all the edges of another rectangular box is 20cm, and the length is twice the width, find the width which would maximise the volume of this box.

(Marks: 5)

3 Consider the problem of wrapping a box with dimensions x, y, z centimetres with paper as shown in the following diagram.

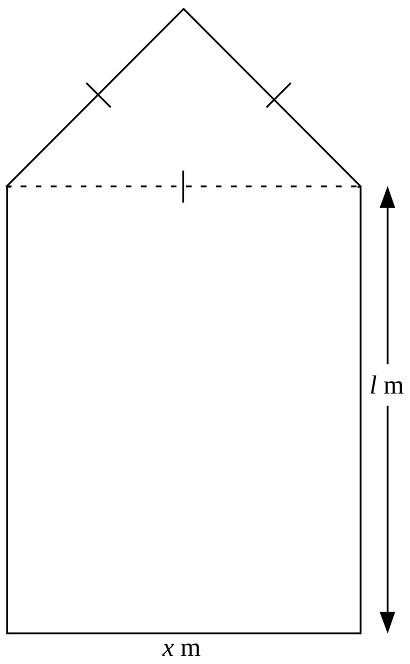


Assume that the paper to be used is just sufficient to wrap the box with the shaded rectangles cut out and removed.

If the volume of the box and z is also fixed, find expressions for x and y, in terms of V and z, so that the area of wrapping paper used is a minimum.

(Marks: 5)

4 One of the main gates in the city of Shtam has a composite shape of a rectangle and an equilateral triangle as shown.



a Write an expression for the perimeter, P, of the gate in terms of x and l.

---

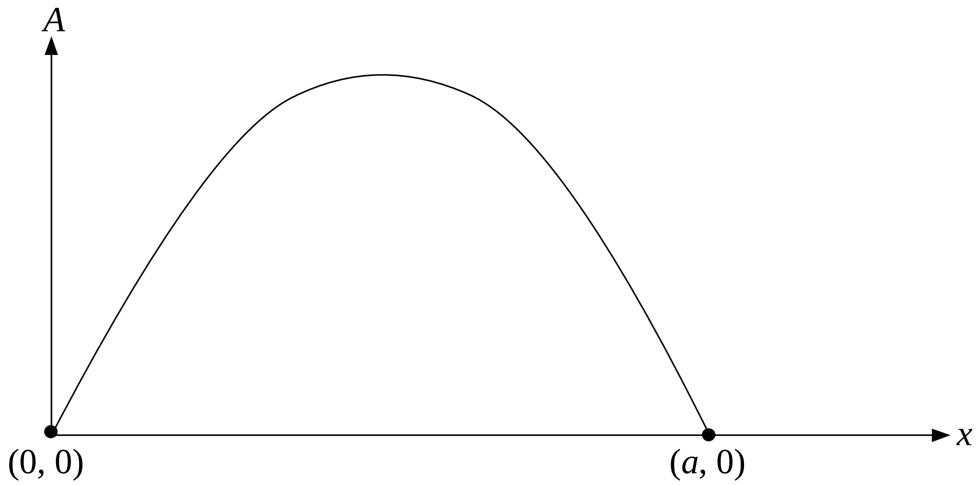
b Write an expression for the area, A, of the gate in terms of x and l.

---

c Initially assume that the perimeter is fixed (a constant value). Show that the area, $A\text{m}^2$, is given by $A = \left(\frac{\sqrt{3}-6}{4}\right)x^2 + \frac{P}{2}x$.

---

d The graph of function A, is shown in the diagram below.



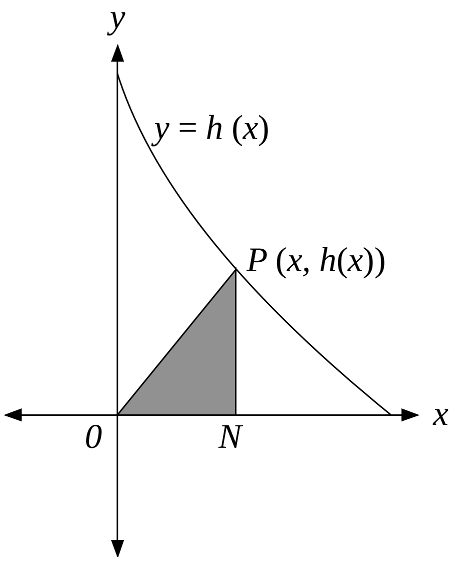
Determine the value of a (the x value of the x-intercept) in terms of P.

e Now assume that the area is fixed (a constant value). Show that $P = \dfrac{2A}{x} + 3x - \dfrac{\sqrt{3}}{2}x$.

f From part **e**, find, using calculus, the value of x (in terms of A) that produces the maximum perimeter, given a fixed area.

(Marks: 1 + 2 + 1 + 2 + 2 + 3 = 11)

5 Let $h:[0,1]\to R,\, h(x)=1-x^{\frac{2}{3}}$. The graph of *f* is shown below.



The right-angled triangle *NOP* has vertex *N* on the *x*-axis, and vertex *O* at the origin. The vertex *P* lies on the graph of *h* and has coordinates $(x, h(x))$ as shown.

a Show that the area *A*, of triangle *NOP* is $\frac{1}{2}x-\frac{1}{2}x^{\frac{5}{3}}$.

____

____

____

b Find the value of *x* for which *A* is a maximum.

____

____

____

____

____

____

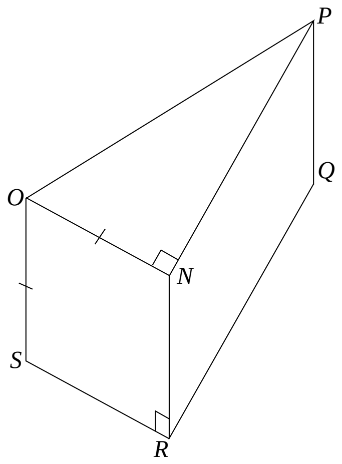
c Find the maximum area of triangle *NOP*.

Give your answer in the form $\frac{a\sqrt{b}}{c}$ where *a*, *b* and *c* are positive integers.

____

____

d If this triangle, *NOP*, is the cross-section of a triangular prism as shown below, and $OS = ON$, find the value of x so that the volume of the prism is maximised.



(Marks: 1 + 2 + 1 + 3 = 7)

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) \quad nx^{n \quad 1}$	$\int x^n dx \quad \frac{1}{n+1}x^{n+1}+c,\ n \neq \quad 1$
$\frac{d}{dx}\left((ax+b)^n\right) \quad an(ax+b)^{n \quad 1}$	$\int (ax+b)^n dx \quad \frac{1}{a(n+1)}(ax+b)^{n+1}+c, n \neq \quad 1$
$\frac{d}{dx}\left(e^{ax}\right) \quad ae^{ax}$	$\int e^{ax} dx \quad \frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right) \quad \frac{1}{x}$	$\int \frac{1}{x} dx \quad \log_e(x)+c,\ x>0$
$\frac{d}{dx}\left(\sin(ax)\right) \quad a\ \cos(ax)$	$\int \sin(ax) dx \quad \ \frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)= \quad a\ \sin(ax)$	$\int \cos(ax) dx \quad \frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)} \quad a\ \sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) \quad u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) \quad \frac{v\frac{du}{dx} \quad u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x - \mu)^2\, p(x)$
continuous	$\Pr(a < X < b) \quad \int_a^b f(x)dx$	$\mu \quad \int_{\infty}^{\infty} x\, f(x)dx$	$\sigma^2 \quad \int_{\infty}^{\infty} (x \quad \mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET