Name: ____________________________

Teacher: ___________________________

School No: _____________

Scotch College

Department of Mathematics

2018

Mathematical Methods

Unit 3 SAC 1c

Application Task

CAS calculator and scientific calculator allowed

No notes allowed

Date: 3:30pm, Thursday 7 June 2018

Time: 45 minutes

Marks: **32**

Instructions

Answer **all** questions in the spaces provided.

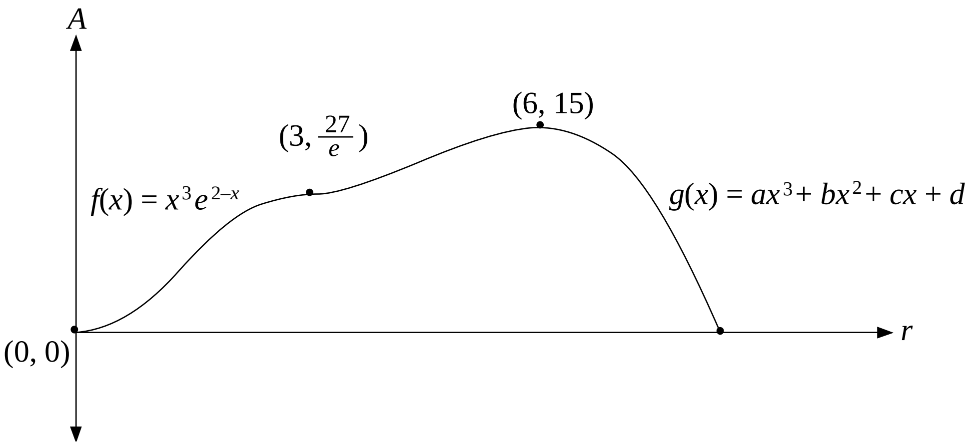
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this task are **not** drawn to scale.

1 Consider the function $f(x) = x^3 e^{2-x}$.

a Show using calculus that $f\left(x\right)$ has a local maximum at $(3, \frac{27}{e})$. Justify it is a local maximum.

b Consider a second curve g , where $g(x) = ax^3 + bx^2 + cx + d$. This curve is smoothly connected to function f at the point $\left(3, \frac{27}{e}\right)$ and has a second stationary point at $(6,15)$.

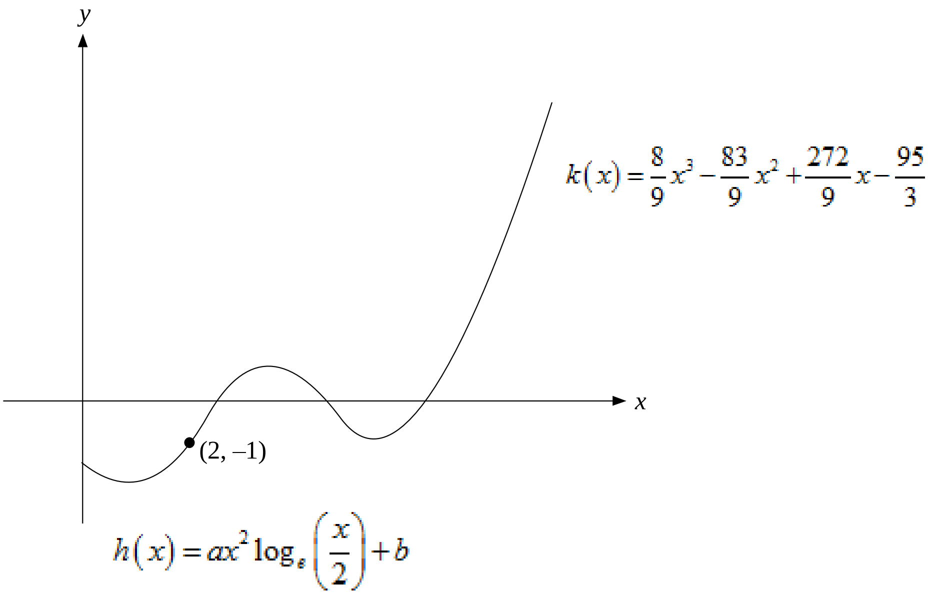


i Write four equations relating to function g from the information provided.

ii Determine the values of a, b, c and d. Give your answers to three decimal places.

(Marks: 4 + (4 + 1) = 9)

2 Two curves $h(x)$ and $k(x)$ are connected smoothly at $(2,-1)$ as shown in the diagram below.



a Determine the value of a and b.

b Show the equation of the tangent to $k(x)$ at $x=3$ is given by $9y+10x=30$.

c Show the equation of the normal to $k(x)$ at $x = 5$ is given by $14y + 3x = 15$

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d Find the acute angle, to the nearest degree, between the two lines in **b** and **c**.

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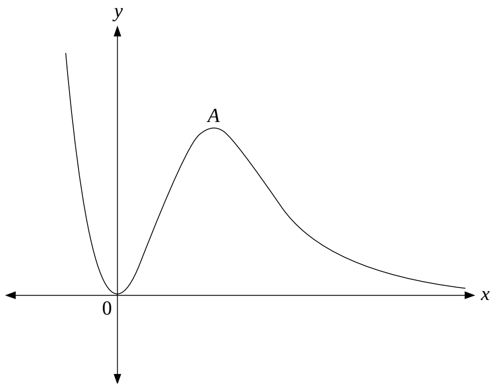
(Marks: 4 + 3 + 3 + 3 = 13)

3 The function p is defined as $p: \mathbb{R} \rightarrow \mathbb{R}$, where $p(x) = x^m e^{(n-x)}$ where m and n are positive integers. The graph of $y = p(x)$ is shown below.



a Let $m = 2$ and $n = 3$.

i Find the value of x, for $x < 0$, such that $p(x) = 1$. Give your answer correct to three decimal places.

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ii State the values of x, correct to three decimal places, such that $p(x) > 1$.

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b **i** State the transformations from the graph of $y = x^2e^{3-x}$ to the graph of

$y_1 = \left(\frac{x-h}{2}\right)^2 e^{\frac{6+h-x}{2}}$ where h is constant.

ii State the coordinates of the stationary points of y_1, in terms of h.

c For $p:\mathbb{R} \to \mathbb{R}$, $p(x) = x^m e^{(n-x)}$ and m and n are unknown.

i Show that the stationary points occur at $x = 0$ and $x = m$.

Continued on next page….

ii Show that the only tangents drawn to the curve that pass through the origin are at $x = 0$ and $x = m - 1$.

(Marks: (1 + 1) + (2 + 2) + (2 + 2) = 10)

END OF TEST COMPONENT SAC 1c

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) \quad nx^{n \quad 1}$		$\int x^n dx \quad \frac{1}{n+1}x^{n+1}+c, \; n \neq \quad 1$	
$\frac{d}{dx}\left((ax+b)^n\right) \quad an(ax+b)^{n \quad 1}$		$\int (ax+b)^n dx \quad \frac{1}{a(n+1)}(ax+b)^{n+1}+c, n \neq \quad 1$	
$\frac{d}{dx}\left(e^{ax}\right) \quad ae^{ax}$		$\int e^{ax} dx \quad \frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right) \quad \frac{1}{x}$		$\int \frac{1}{x} dx \quad \log_e(x)+c, \; x>0$	
$\frac{d}{dx}\left(\sin(ax)\right) \quad a \; \cos(ax)$		$\int \sin(ax)dx \quad \frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right) = \quad a \; \sin(ax)$		$\int \cos(ax)dx \quad \frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} \quad a \; \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) \quad u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) \quad \frac{v\frac{du}{dx} \quad u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \vert B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x - \mu)^2\, p(x)$
continuous	$\Pr(a < X < b) \quad \int_a^b f(x)dx$	$\mu \quad \int_{\infty}^{\infty} x\, f(x)dx$	$\sigma^2 \quad \int_{\infty}^{\infty} (x \quad \mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET