	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
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Teacher's Name

Scotch College

MATHEMATICAL METHODS

U3-SAC 1a – Application Task: Project

Date of distribution: Monday 27th May 2019

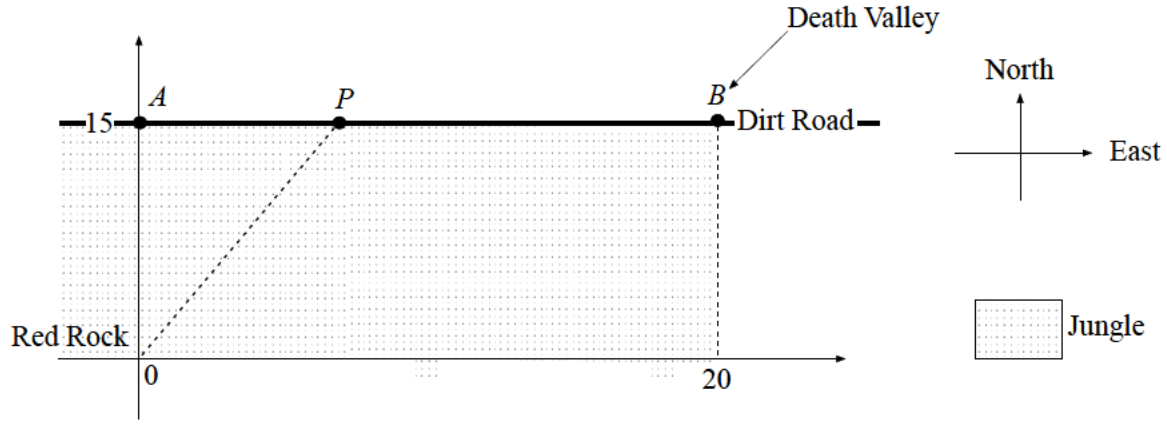
Due date: Tuesday 11th June 2019

Task Sections	Marks	Your Marks
Extended Response Questions	60	
Total Marks	60	

General Instructions
• Answer all questions in the spaces provided. • In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. • In questions where more than one mark is available, appropriate working must be shown. • Unless otherwise indicated, the diagrams in this task are not drawn to scale.
Allowed Materials
• A scientific calculator and a CAS calculator. • Any notes or references.
At the end of the task
• Submit the task to your teacher by the due date.

Question 1 (3 marks)

As part of a military training exercise, a group of commandos is required to hike from Red Rock to Death Valley as depicted on the diagram below.



Death Valley is located 15 km north and 20 km east of Red Rock. There is a dirt road running east-west that is 15 km north of Red Rock. The commandos are able to hike at a speed of 2 km/h through the jungle to a point P on the track and then at a speed of $2n$ km/h along the dirt road until they reach Death Valley.

The commandos wish to reach Death Valley in the shortest time possible. You must help the commandos by calculating the point P along the dirt road they should aim to walk in order to minimise the time they take to reach Death Valley.

For the purposes of your calculations let the distance $AP = x$ *km*.

a. Find as an expression in terms of x the distance

i. OP 1 mark

ii. PB 1 mark

b. Explain why it should be assumed that $n > 1$. 1 mark

Question 2 (12 marks)

For Question 2, use $n = 4$.

a. Find an expression in terms of x for the total time in hours, $T(x)$, it will take to hike from Red Rock to Death Valley via point P on the dirt road.

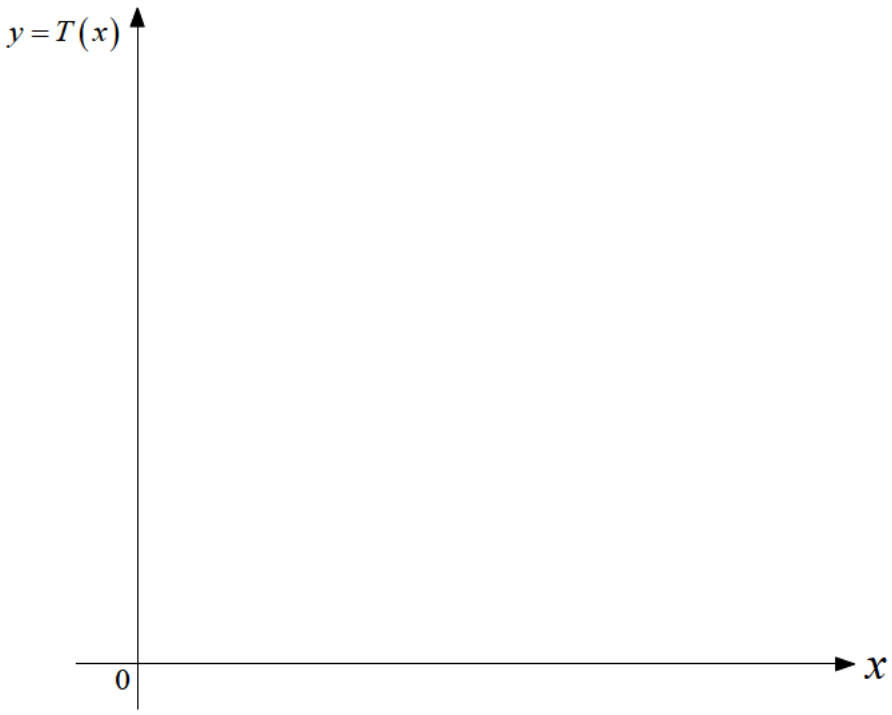
Give an appropriate domain for $T(x)$. 2 marks

b. **i.** Use calculus to show that $T'(x) = \dfrac{x}{2\sqrt{225 + x^2}} - \dfrac{1}{8}$ 1 mark

ii. Hence show, by using calculus, that the function $T(x)$ has a stationary point at $x = \sqrt{15}$ 3 marks

iii. Find the corresponding time, to the nearest minute, and prove that it is the minimum time in which the commandos can arrive at Death Valley. 3 marks

c. Sketch the graph of $y = T(x)$ over an appropriate domain. Label key features correct to 2 decimal places. 3 marks

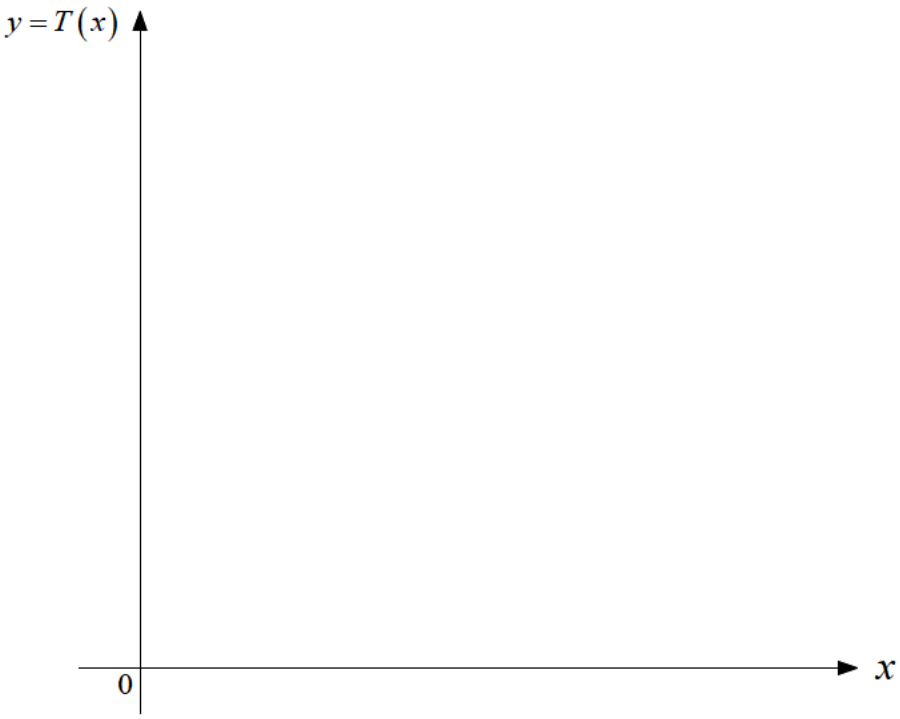


Question 3 (6 marks)

For Question 3, use $n = 1.2$

a. Find an expression in terms of x for the total time in hours, $T(x)$, it will take to hike from Red Rock to Death Valley via point P on the dirt road. 1 mark

b. Sketch the graph of $y = T(x)$ over an appropriate domain. Label key features correct to 2 decimal places. 2 marks



c. **i.** Find $T'(x)$ 1 mark

ii. Hence explain that $T(x)$ is a strictly decreasing function over the domain for which the model is valid. 1 mark

d. State the minimum possible time for the Commandos to hike from Red Rock to Death Valley via point P on the dirt road if $n = 1.2$ 1 mark

Question 4 (11 marks)

For Question 4, n is not specified

a. Find an expression in terms of x and n for the total time, $T(x)$, it will take for the commandos to hike from Red Rock to Death Valley via point P on the dirt road. 1 mark

b. Use calculus to show the route which uses the least time occurs when

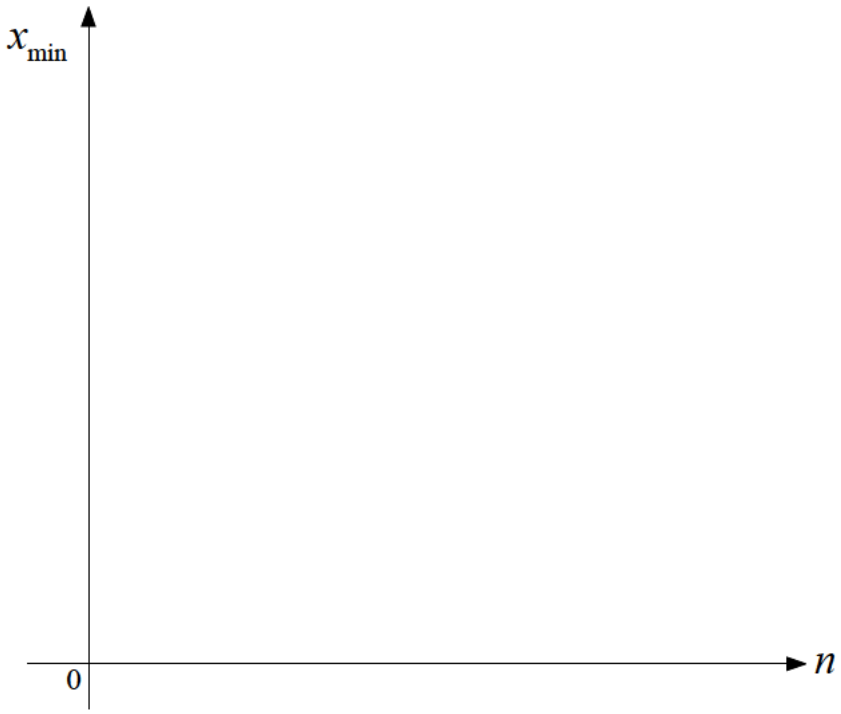
$$x = \frac{15}{\sqrt{n^2 - 1}}$$

Verify this result by considering $n = 4$

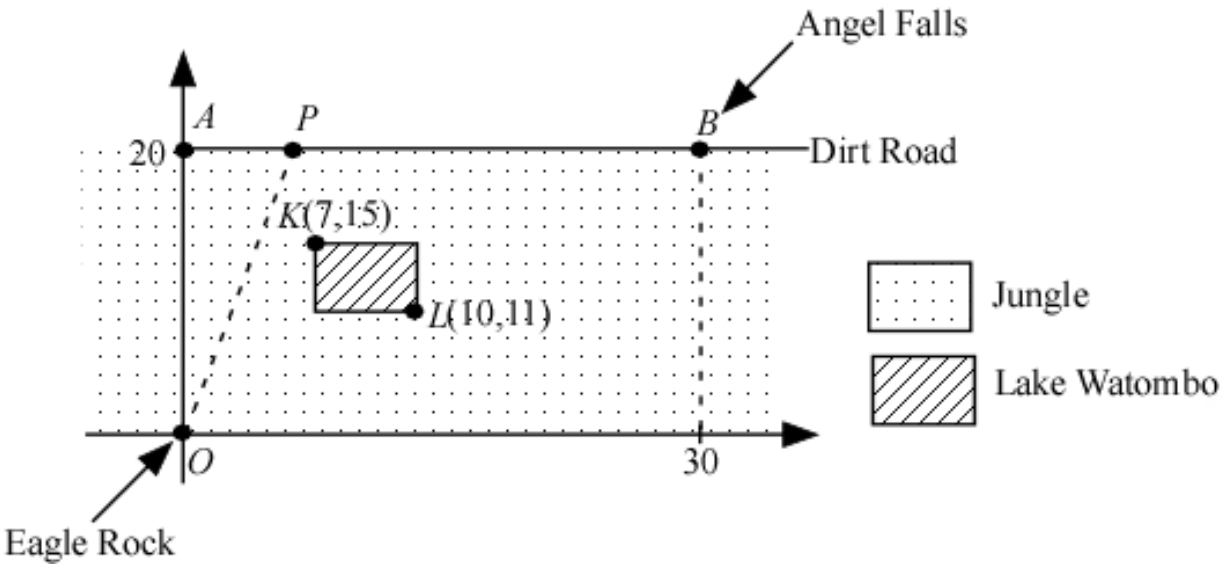
- It will be assumed that this value of x corresponds to a minimum turning point for the graph $y = T(x)$ 5 marks

c. Find the possible values of n in order that the minimum turning point of the graph $y = T(x)$ occurs within the domain for which the model is valid. Explain how this result is consistent with what you observed in Questions 2 and 3. 2 marks

d. With the aid of a graphics calculator, sketch a graph of $x_{\min}$ against n for $n > 1$, where $x_{\min}$ is the x value corresponding to the minimum turning point for the graph $y = T(x)$. Be careful to consider the restriction on $x_{\min}$. Describe what the graph indicates in relation to the hike from Red Rock to Death Valley. 3 marks



During the next stage of the same military training exercise, the group of commandos are now required to hike from Eagle Rock to Angel Falls as depicted on the diagram below.

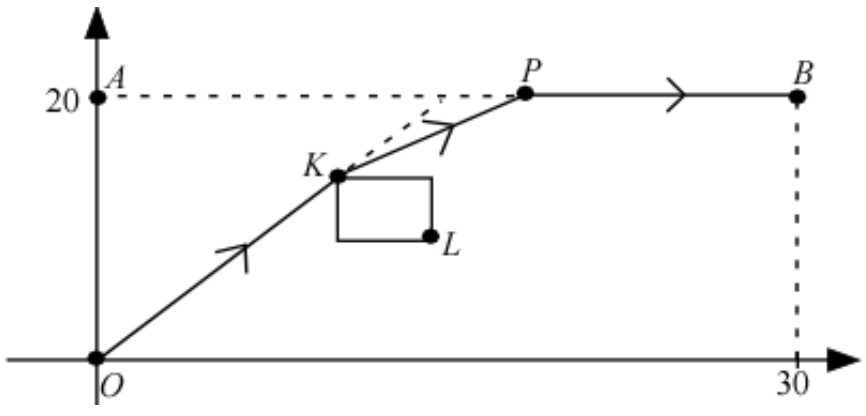


Angel Falls is located $20\ km$ north and $30\ km$ east of Eagle Rock. There is a dirt road running east-west that is 20 km north of Eagle Rock. The commandos can hike at a speed of 3 *km/h* through the jungle to a point *P*, and then at a speed of 6 *km/h* along the dirt road until they reach Angel Falls. However, Lake Watombo is located within the jungle as shown on the above diagram. Any hike route that the commandos choose must not pass through any part of Lake Watombo. Lake Watombo is rectangular in shape.

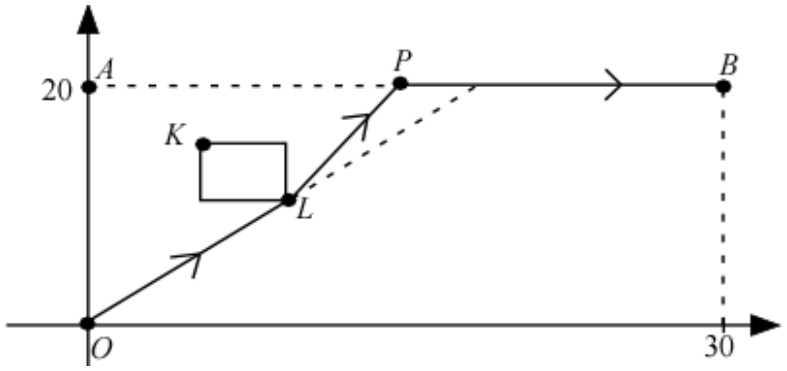
The commandos wish to reach Angel Falls in the shortest possible time. Help them by calculating the location of the point *P* on the dirt road in order to minimise the time they take to reach Angel Falls.

The commandos may walk from Eagle Rock to Angel Falls in one of the following ways:

- In a straight line from *O* to *P* through the jungle (providing they miss Lake Watombo), then along the dirt road to *B*.
- They may walk from *O* to *K* (if the lake is in their pathway), then deviate to the right of the straight line path to *P* through the jungle and along the dirt road to *B,* as depicted below.



- They may walk from O to L (if the lake is in their pathway), then deviate to the left of the straight line path to P through the jungle and along the dirt road to B, as depicted below.



For the purposes of your calculations let the distance $AP = x$ km. The point K is one corner of of Lake Watombo at $(7, 15)$ and the point L is another corner of Lake Watombo at $(10, 11)$.

Question 5 (3 marks)

a. Find the distance OK 1 mark

b. Find an expression in terms of x, the distance:

i. KP 1 mark

ii. PB 1 mark

Question 6 (14 marks)

Depending on the location of the point P, the commandos can hike in a straight line from the point O to the point P avoiding Lake Watombo. Wherever possible, the commandos will follow a straight line path.

a. Determine an expression in terms of x for the straight line distance OP 1 mark

b. Hence write down the rule for $f(x)$ the total time taken to hike in a straight line from O to P then along the road to B assuming the hikers miss Lake Watombo. 1 mark

c. Use calculus to find the value of x for which $f(x)$ is a minimum where $x \in \mathbb{R}$ 4 marks

d. Find the value of x when the line segment OP passes through the point

i. K 2 marks

ii. L 2 marks

e. Hence write down the values of x for which $f(x)$ is valid. 2 marks

f. For values of x for which $f(x)$ is valid in the model, find the exact value of x that corresponds to the route which uses the least time when the commandos walk in a straight line directly from O to the point P. 2 marks

Question 7 (3 marks)

The commandos may walk from Eagle Rock to Angel Falls in one of the following ways:

- In a straight line from O to P through the jungle (providing they miss Lake Watombo), then along the dirt road to B. The total time for this path is given by $f(x)$.
- They may walk from O to K (if Lake Watombo is in their pathway), then deviate to the right of the straight line path to P through the jungle and along the dirt road to B. The total time for this path is given by $g(x)$.
- They may walk from O to L (if Lake Watombo is in their pathway), then deviate to the left of the straight line path to P through the jungle and along the dirt road to B. The total time for this path is given by $h(x)$.

a. Write down the function $g(x)$. 2 marks

In a similar manner to **part a** the function $h(x)$ can be found to be:

$$h(x)=\frac{1}{3}\left(\sqrt{221}+\sqrt{x^2-20x+181}\right)+\frac{30-x}{6}$$

b. Solve for $g(x)=h(x)$ for x, correct to 4 decimal places. 1 mark

Question 8 (3 marks)

The function needed to determine the time taken to walk from Eagle Rock to Angel Falls using one of the three pathways described in Question 7 is a hybrid function.

The hybrid function can be written in the form:

$$T(x)=\begin{cases} f(x) & \textit{where} \quad 0 \le x \le b \text{ or } \mathrm{d} \le x \le 30 \\ g(x) & \textit{where} \quad b < x < c \\ h(x) & \textit{where} \quad c \le x < d \end{cases}$$

where $f(x)$ is the function found in Question 6 **part b,** $g(x)$ is the function found in Question 7 **part a** and $h(x)$ is given in **Question 7**. Write out the hybrid function fully (with *c* correct to 4 decimal places).

Question 9 (5 marks)

Use calculus to find the exact value of *x* which corresponds to the route that uses the least time when the commandos walk to the point *P* via the point *L*.

END OF SAC 1a

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)$	$nx^{n\ 1}$	$\int x^n dx$	$\frac{1}{n+1}x^{n+1}+c,\ n\neq\ 1$
$\frac{d}{dx}\left((ax+b)^n\right)$	$an(ax+b)^{n\ 1}$	$\int (ax+b)^n dx$	$\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq\ 1$
$\frac{d}{dx}\left(e^{ax}\right)$	ae^{ax}	$\int e^{ax}dx$	$\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}(\log_e(x))$	$\frac{1}{x}$	$\int \frac{1}{x}dx$	$\log_e(x)+c,\ x>0$
$\frac{d}{dx}(\sin(ax))$	$a\ \cos(ax)$	$\int \sin(ax)dx$	$\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}(\cos(ax))=$	$a\ \sin(ax)$	$\int \cos(ax)dx$	$\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}(\tan(ax))=\frac{a}{\cos^2(ax)}$	$a\ \sec^2(ax)$		
product rule	$\frac{d}{dx}(uv)\quad u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)\quad \frac{v\frac{du}{dx}\ \ u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		