	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

Unit 3-SAC 1b – Application Task: Test VERSION 2

Tuesday 11th June 2019

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	30	

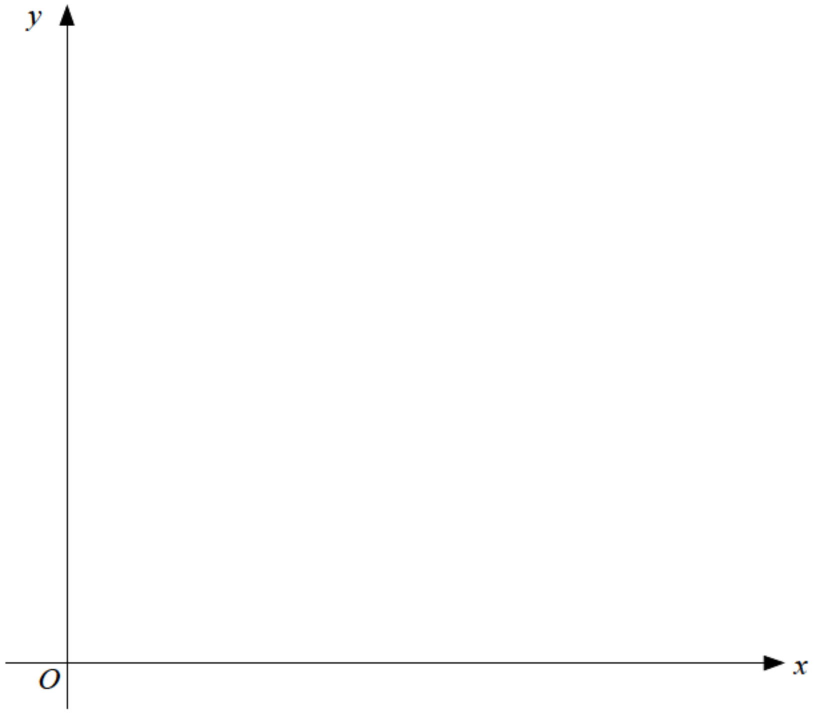
General Instructions
• Answer all questions in the spaces provided. • In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. • In questions where more than one mark is available, appropriate working must be shown. • Unless otherwise indicated, the diagrams in this task are not drawn to scale.
Allowed Materials
• Calculators are not allowed • Notes and/or references are not allowed
At the end of the task
• Ensure you cease writing upon request.
Electronic Devices
Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (7 marks)

Consider the two functions:

$f:[0,\infty)\to\mathbb{R}$, where $f(x)=x+4$ and $g:[0,\infty)\to\mathbb{R}$, where $g(x)=\sqrt{x}$

a. Sketch on the axes below the graphs of both $y=f(x)$ and $y=g(x)$ 2 marks



b. Write down the rule for $h(x)$, the vertical distance between the two graphs.

Give an appropriate domain for $h(x)$. 1 mark

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c. Calculate the value of x which corresponds to the minimum vertical distance between the two functions over the appropriate domain. 3 marks

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d. Hence evaluate the minimum vertical distance between the two functions $f(x)$ and $g(x)$. 1 mark

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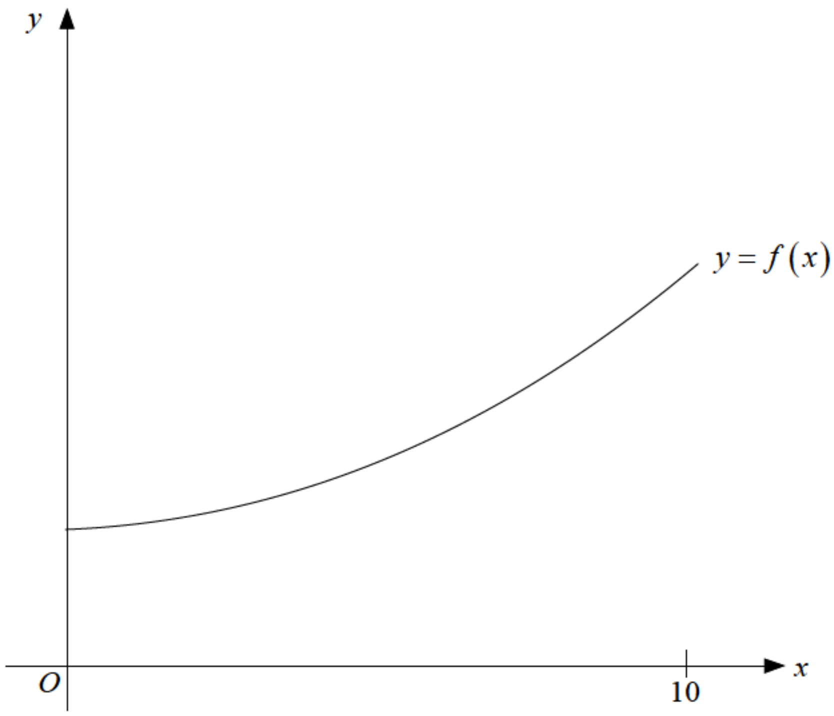
Question 2 (7 marks)

Consider the two functions:

$f:[0,10]\to\mathbb{R}$, where $f(x)=\dfrac{\sqrt{x^2+16}}{3}$ and $g:[0,10]\to\mathbb{R}$, where $g(x)=\dfrac{10-x}{5}$

The graph of $y=f(x)$ is sketched on the axes below.



a. **i.** On the above axes, sketch $y=g(x)$. 1 mark

ii. On the above axes, sketch $y=h(x)$, where $h(x)=\dfrac{\sqrt{x^2+16}}{3}+\dfrac{10-x}{5}$ for $x\in[0,10]$ 1 mark

b. Use calculus to show that $h'(x)=\dfrac{x}{3\sqrt{x^2+16}}-\dfrac{1}{5}$ 1 mark

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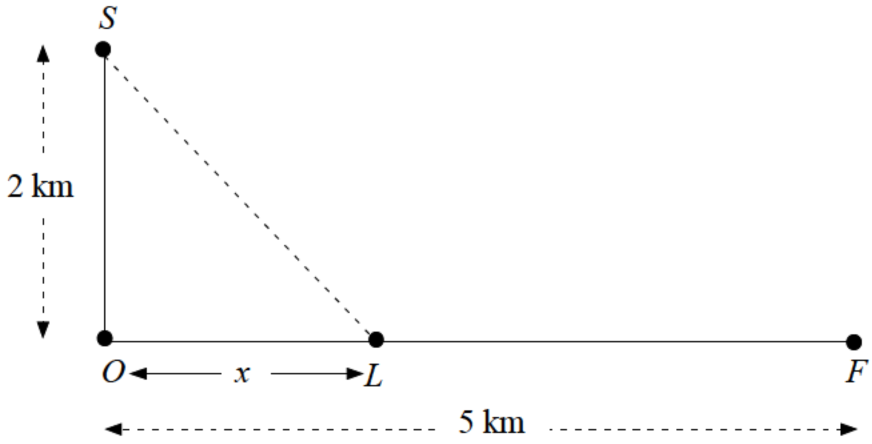
c. Calculate the value of x that corresponds to any stationary points for $h(x)$. 3 marks

d. Hence, find the minimum value of $h(x)$ for $x \in [0,10]$ 1 mark

Question 3 (7 marks)



The point S is an island 2 km offshore from the point O which is located on a straight sandy stretch of beach, as shown in the diagram above. The point F is on the beach, 5 km from the point O. Competitors race from the island to the finish point at F by rowing in a straight line to some point L along the beach and then running along the beach to F.

A particular competitor rows at 4 km/h and runs at 8 km/h.

a. Show that if the distance OL is x km, the time taken by this competitor to complete the race (in hours) is given by: 2 marks

$$T(x)=\frac{\sqrt{x^2+4}}{4}+\frac{5-x}{8}$$

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b. Show that the time taken by this competitor to complete the race has its minimum value when $x=\frac{2\sqrt{3}}{3}$ 3 marks

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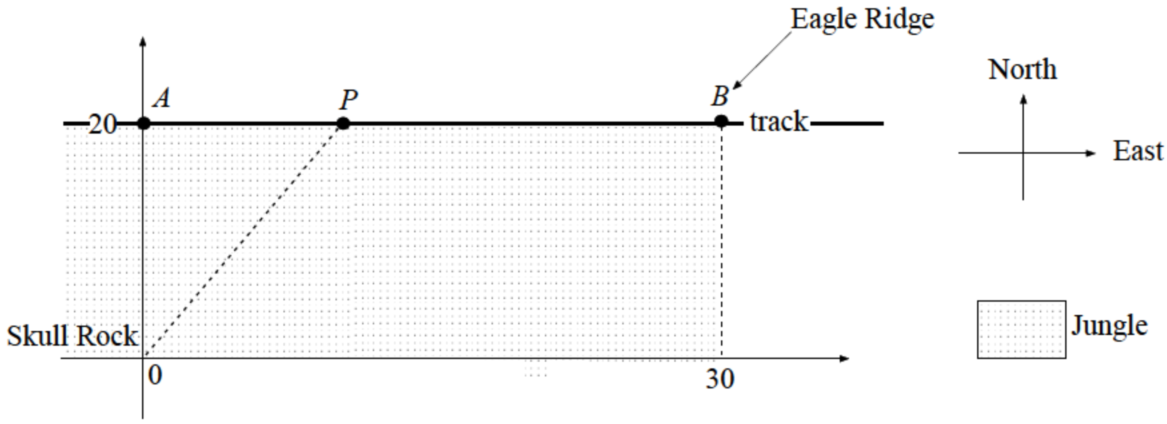
c. Hence state the competitor's minimum race time.

Write your answer in the form $\frac{1}{8}\left(a\sqrt{3}+b\right)$ 2 marks

Question 4 (9 marks)



Eagle Ridge is located 20 km north and 30 km east of Skull Rock. There is a track that runs in an east-west direction that is 10 km north of Skull Rock. A group of explorers are able to hike at a speed of 3 km/h through the jungle to a point P on the track and then at a speed of $3n$ km/h along the track until they reach Eagle Ridge. The explorers wish to reach Eagle Ridge in the shortest time possible.

For the purposes of your calculations let the distance $AP = x\,km$.

a. Explain why it should be assumed that $n > 1$. 1 mark

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b. Find an expression in terms of x for the total time in hours, $T(x)$, it will take to hike from Skull Rock to Eagle Ridge via point P on the track.

Give an appropriate domain for $T(x)$. 1 mark

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c. **i.** Use calculus to show that $T'(x) = \frac{x}{3\sqrt{400 + x^2}} - \frac{1}{3n}$ 2 marks

ii. Use calculus to show the route which uses the least time to travel from Skull Rock to Eagle Ridge occurs when $x = \frac{20}{\sqrt{n^2 - 1}}$ 3 marks

iii. Find the possible values of n in order that the minimum turning point of the graph $y = T(x)$ occurs within the domain for which the model is valid. 2 marks

END OF SAC 1b

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) \quad nx^{n \quad 1}$	$\int x^n dx \quad \frac{1}{n+1}x^{n+1}+c, \ n \neq \quad 1$
$\frac{d}{dx}\left((ax+b)^n\right) \quad an(ax+b)^{n \quad 1}$	$\int (ax+b)^n dx \quad \frac{1}{a(n+1)}(ax+b)^{n+1}+c, n \neq \quad 1$
$\frac{d}{dx}\left(e^{ax}\right) \quad ae^{ax}$	$\int e^{ax} dx \quad \frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right) \quad \frac{1}{x}$	$\int \frac{1}{x} dx \quad \log_e(x)+c, \ x>0$
$\frac{d}{dx}\left(\sin(ax)\right) \quad a \ \cos(ax)$	$\int \sin(ax) dx \quad \quad \frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)= \quad a \ \sin(ax)$	$\int \cos(ax) dx \quad \frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)} \quad a \ \sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) \quad u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) \quad \frac{v\frac{du}{dx} \quad u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		