	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Scotch College

Teacher's Name

MATHEMATICAL METHODS

U4-SAC 1a – Application Task: Project

Date of distribution: Monday 5th August 2019

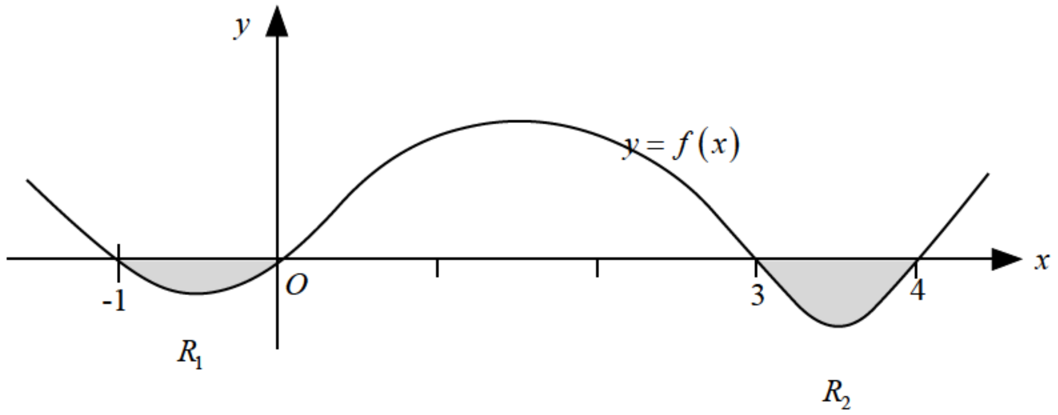
Due date: Thursday 15th August 2019

Task Sections	Marks	Your Marks
Extended Response Questions	72	
Total Marks	72	

General Instructions
• Answer all questions in the spaces provided. • In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. • In questions where more than one mark is available, appropriate working must be shown. • Unless otherwise indicated, the diagrams in this task are not drawn to scale.
Allowed Materials
• A scientific calculator and a CAS calculator. • Any notes or references.
At the end of the task
• Submit the task to your teacher by the due date.

Question 1 (2 marks)

The diagram shows the graph of $y = f(x)$ which intercepts at $x = -1, 0, 3$ and 4.



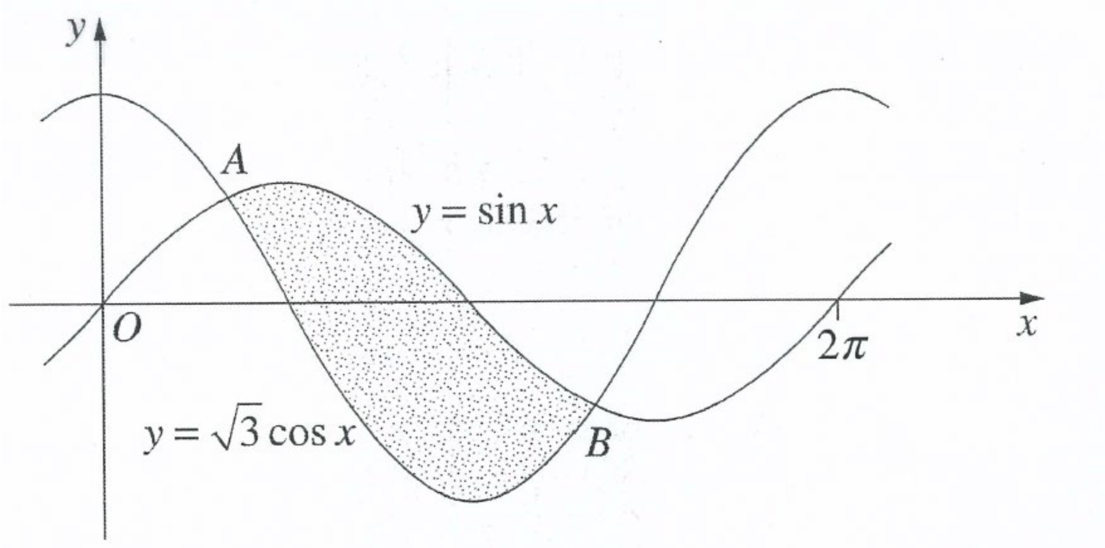
The area of shaded region R_1 is 2.

The area of shaded region R_2 is 3.

It is given that $\int_0^4 f(x)\,dx = 10$.

What is the value of $\int_{-1}^3 f(x)\,dx$?

Question 2 (4 marks)



The diagram shows the graphs of $y = \sqrt{3}\cos x$ and $y = \sin x$. The first two points of intersection to the right of the y-axis are labelled A and B.

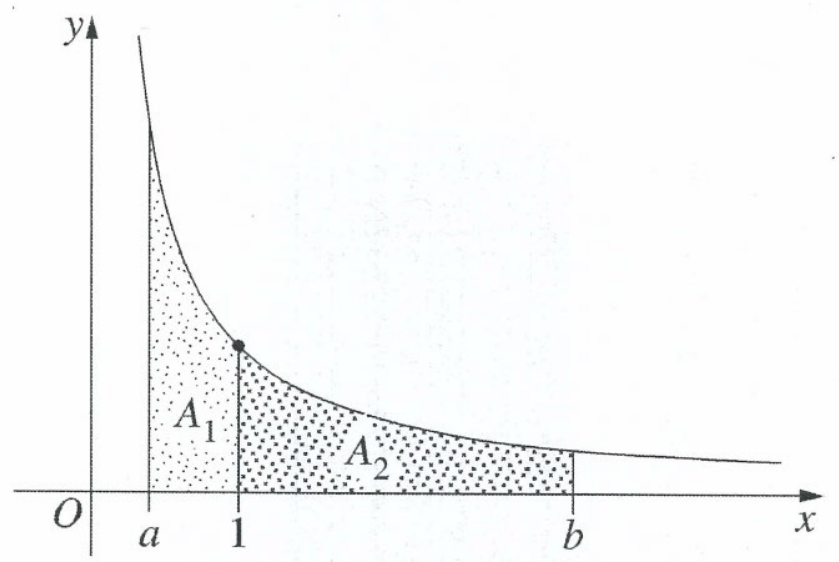
a. Solve the equation $\sqrt{3}\cos x = \sin x$ to find the x-coordinates of A and B. 2 marks

b. Find the area of the shaded region in the diagram. 2 marks

Question 3 (4 marks)

The diagram shows the curve $y = \frac{1}{x}$, for $x > 0$.

The area under the curve between $x = a$ and $x = 1$ is A_1. The area under the curve between $x = 1$ and $x = b$ is A_2.



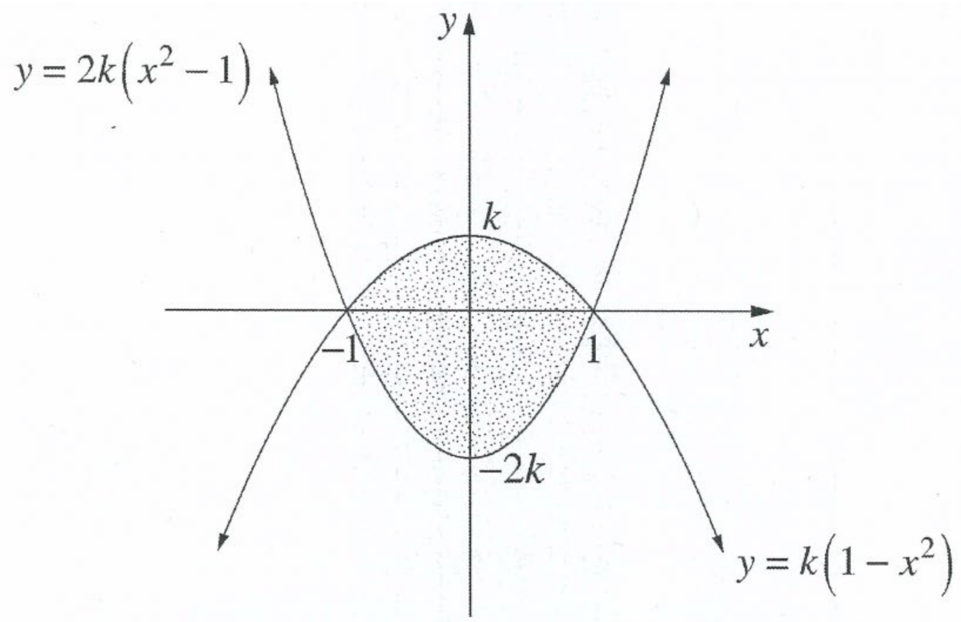
The areas A_1 and A_2 are each equal to 1 square unit.

Find the values of *a* and *b*.

Question 4 (4 marks)

The shaded region shown is enclosed by two parabolas, each with x-intercepts at $x = -1$ and $x = 1$.

The parabolas have equations $y = 2k\left(x^2 - 1\right)$ and $y = k\left(1 - x^2\right)$, where $k > 0$.



Given that the area of the shaded region is 8, find the value of k. Show all working.

Question 5 (5 marks)

The cross section of a waterway is parabolic. Its depth is 3 metres, and the width across the top of the waterway is 4 metres. When the waterway is one-third full, what is the depth of the water in metres, correct to two decimal places?

Question 6 (4 marks)

Find the value of k for which the line with equation $y = k$ bisects the area enclosed by the curve $y = x^2$ and the lines $y = 9$ and $x = 0$. Write your answer correct to two decimal places.

Question 7 (4 marks)

Find the value of k for which the line with equation $y = kx$ bisects the area enclosed by the curve $4y = 4x - x^2$ and the x-axis. Write your answer correct to four decimal places.

Question 8 (3 marks)

The region bounded by the x-axis and part of the graph of $y = \cos(x)$ between $x = 0$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. Find the value of k, correct to three decimal places, if the area of the region for $0 \le x \le k$ is one third of the area of the region for $k \le x \le \frac{\pi}{2}$.

Question 9 (6 marks)

a. Differentiate $\log_e(\cos x)$ with respect to x. 2 marks

b. Hence, show that $\int_0^{\frac{\pi}{4}} \tan x \, dx = \frac{1}{2}\log_e 2$. 4 marks

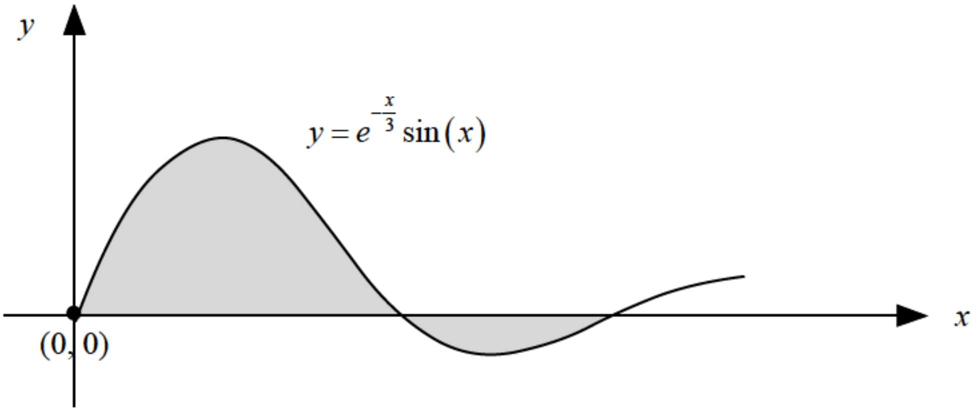
Question 10 (9 marks)

a. Let $f: \mathbb{R}^+ \to \mathbb{R}, f(x) = e^{-\frac{x}{3}}\left(m\cos(x) + n\sin(x)\right)$

Given that $f'(x) = e^{-\frac{x}{3}}\sin(x)$, find the constants *m* and *n*. 3 marks

b. Calculate the exact value of $\int_0^{2\pi} e^{-\frac{x}{3}}\sin(x)\,dx$ 3 marks

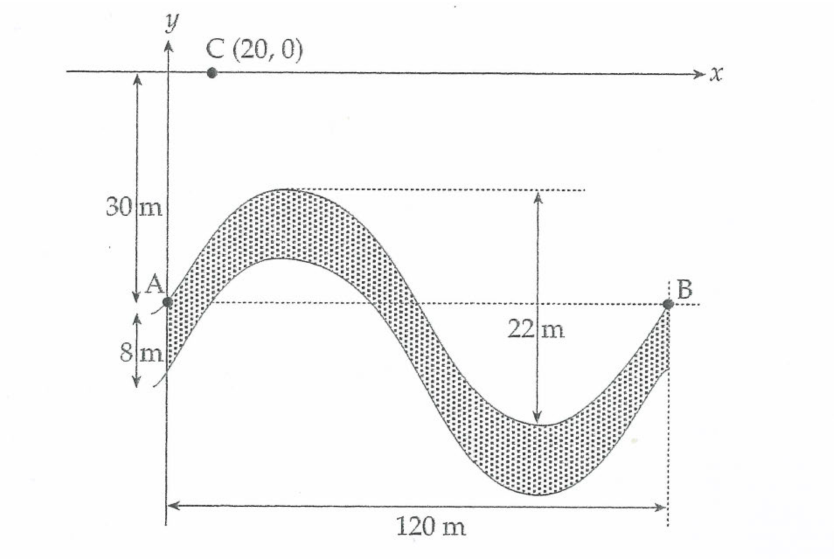
c.



Calculate the area of the shaded region as shown above, correct to 4 decimal places. 3 marks

Question 11 (14 marks) ***All answers in this question should be given to 2 decimal places.***

A layer of ore beneath the ground in outback Australia has surfaces that are sinusoidal in cross section. A mining company has drawn up a rough sketch on a set of axes so that the x-axis represents the surface:



The mining company feels that an appropriate equation to represent the upper layer of the sinusoidal curve, relative to the set of axes is

$$y = a\sin(kx) + d, \qquad 0 \le x \le 120.$$

The layer can be assumed to have everywhere, a vertical depth of 8m.

a. Show that $a = 11, d = -30$ and $k = \frac{\pi}{60}$ 3 marks

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b. What is the minimum distance from the surface that the miners will need to drill to reach the ore? 2 marks

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c. How deep must the miners drill when they are at point C, if they must drill through to the lower layer of the ore? 2 marks

Once the miners have drilled at point C and have reached the lower layer, a pipe is to run horizontally until it hits the layer of ore once again.

d. What is the minimum length that this horizontal pipe will need to be? 4 marks

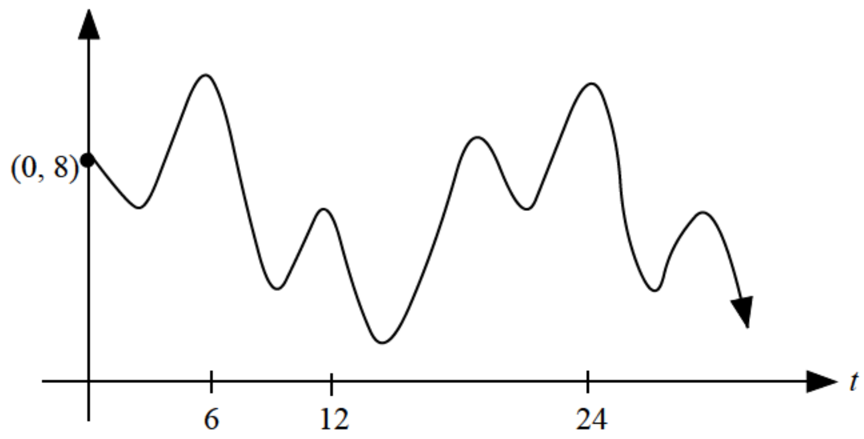
e. Calculate the cross-sectional area of ore that lays above the pipe in **part d.** 3 marks

Question 12 (6 marks)

It is common practice in the building industry to include heating in concrete slabs. Hot water runs through pipes within the concrete slab of a house. The concrete slab absorbs the heat from the water and releases it into the area above. The number of litres/minute of water flowing through the piping over t minutes can be modelled by the rule

$$\frac{dV}{dt} = 2\left(\cos\left(\frac{\pi t}{3}\right) + \sin\left(\frac{\pi t}{9}\right) + 3\right)$$

The graph of this function is shown below:



a. What is the rate of flow of water, correct to 2 decimal places, at

i. 4 minutes — 1 mark

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ii. 8 minutes — 1 mark

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b. State the period of the given function. — 2 marks

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c. Find the volume of water that flows through the pipes during the time period for one entire cycle. — 2 marks

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Question 13 (7 marks)

a. Given that:

$$A(2\sin x+\cos x)+B(2\cos x-\sin x)=7\sin x+11\cos x,$$

find the values of A and B. 2 marks

b. Hence, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} \frac{7\sin x+11\cos x}{2\sin x+\cos x}\,dx=\frac{5\pi}{2}+\log_e 8$$ 5 marks

END OF SAC 1a

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) \quad nx^{n \quad 1}$		$\int x^n dx \quad \frac{1}{n+1}x^{n+1}+c,\ n\neq \quad 1$	
$\frac{d}{dx}\left((ax+b)^n\right) \quad an(ax+b)^{n \quad 1}$		$\int (ax+b)^n dx \quad \frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq \quad 1$	
$\frac{d}{dx}\left(e^{ax}\right) \quad ae^{ax}$		$\int e^{ax} dx \quad \frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right) \quad \frac{1}{x}$		$\int \frac{1}{x} dx \quad \log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right) \quad a\ \cos(ax)$		$\int \sin(ax) dx \quad \frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)= \quad a\ \sin(ax)$		$\int \cos(ax) dx \quad \frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)} \quad a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) \quad u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) \quad \frac{v\frac{du}{dx} \quad u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		