Circle the relevant digits	Scotch Student ID #			
	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

Unit 4-SAC 1b – Application Task: Test

Thursday 15th August 2019

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	35	
Total Marks	35	

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are not allowed
- Notes and/or references are not allowed

At the end of the task

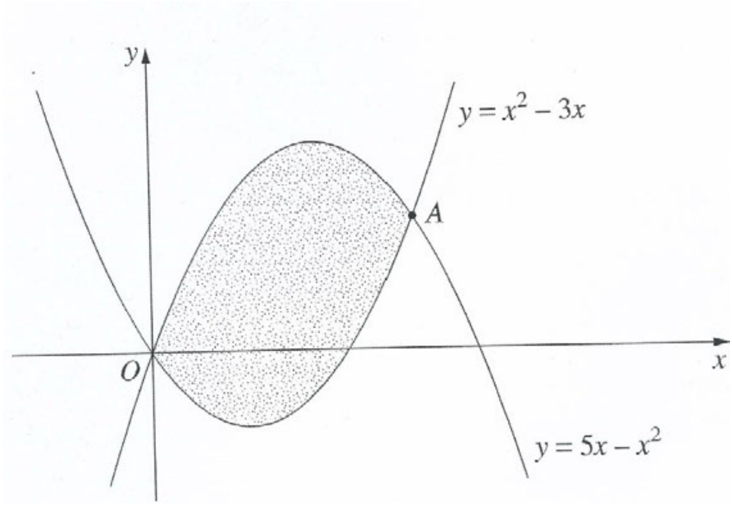
- Ensure you cease writing upon request.

Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (4 marks)

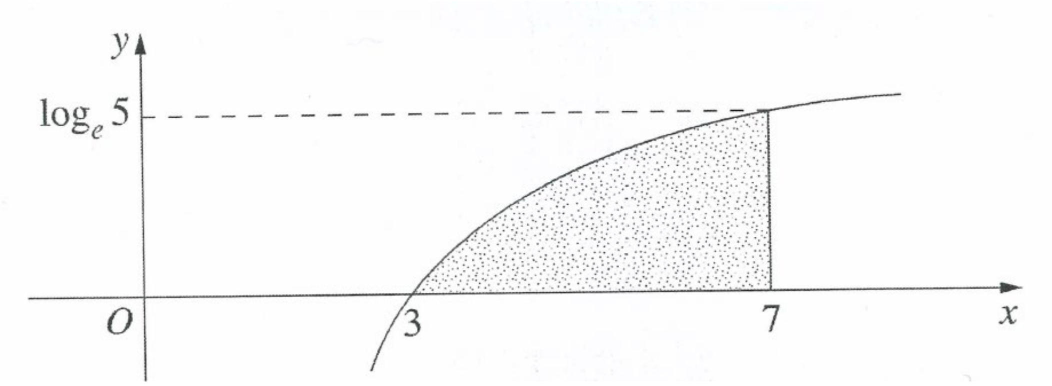
The diagram below shows the parabolas $y = 5x - x^2$ and $y = x^2 - 3x$. The parabolas intersect at the origin O and the point A. The region enclosed between the two parabolas is shaded.



a. Find the x-coordinate of the point A. 2 marks

b. Find the area of the shaded region. 2 marks

Question 2 (4 marks)

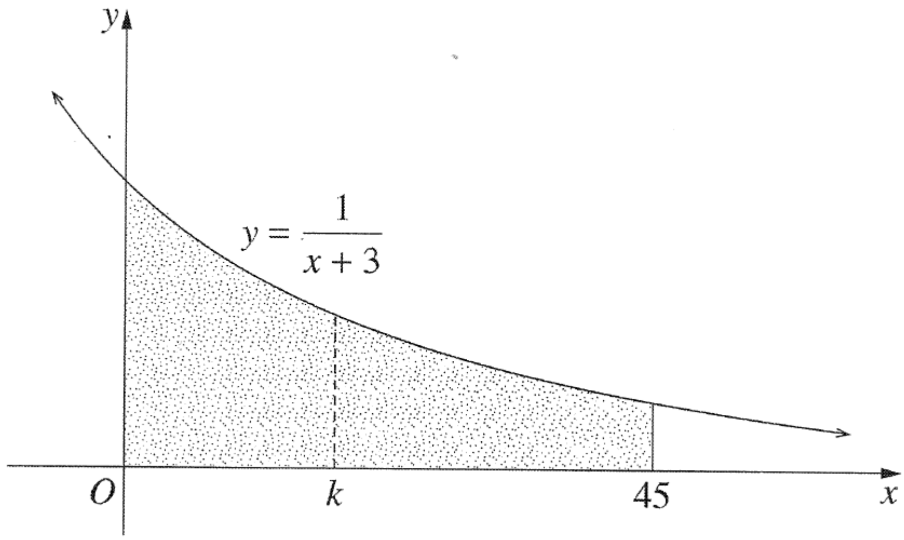


In the diagram above, the shaded region is bounded by the curve $y = \log_e(x-2)$, the x-axis and the vertical line $x = 7$.

Find the exact value of the area of the shaded region.

Question 3 (5 marks)

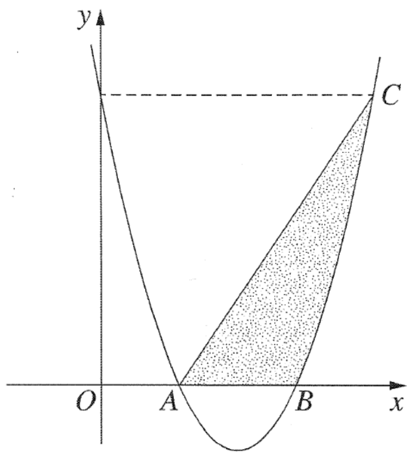
The diagram shows a region bounded by the curve $y = \dfrac{1}{x+3}$ and the lines $x = 0, x = 45$ and $y = 0$. The region is divided into two parts of equal area by the line $x = k$, where k is a positive integer.



What is the value of the integer k, given that the two parts have equal areas?

Question 4 (8 marks)

The diagram below shows the parabola with equation $y = x^2 - 7x + 10$. The parabola intersects the x-axis at points A and B. The point C on the parabola has the same y-coordinate as the y-intercept of the parabola.



a. Find the x-coordinates of points A and B. 2 marks

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b. Find the coordinates of C. 2 marks

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c. Evaluate $\int_0^2 \left(x^2 - 7x + 10\right) dx$. 2 marks

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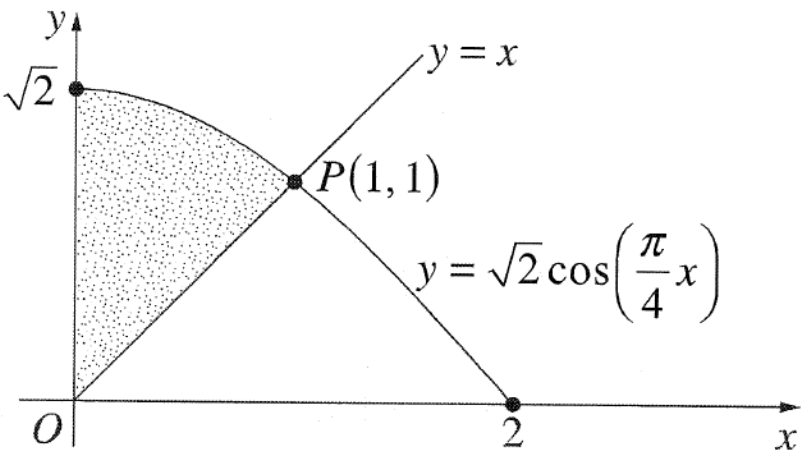
d. Hence, or otherwise, find the area of the shaded region. 2 marks

Question 5 (3 marks)

The curve $y = \sqrt{2}\cos\left(\frac{\pi}{4}x\right)$ meets the line $y = x$ at $P(1,1)$, as shown in the diagram below.

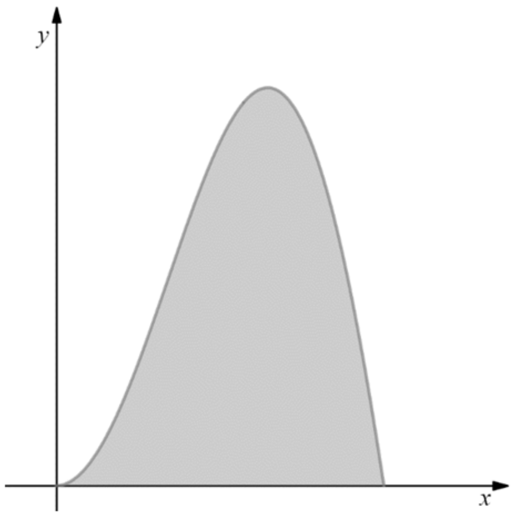


Find the exact value of the shaded area.

Question 6 (11 marks)

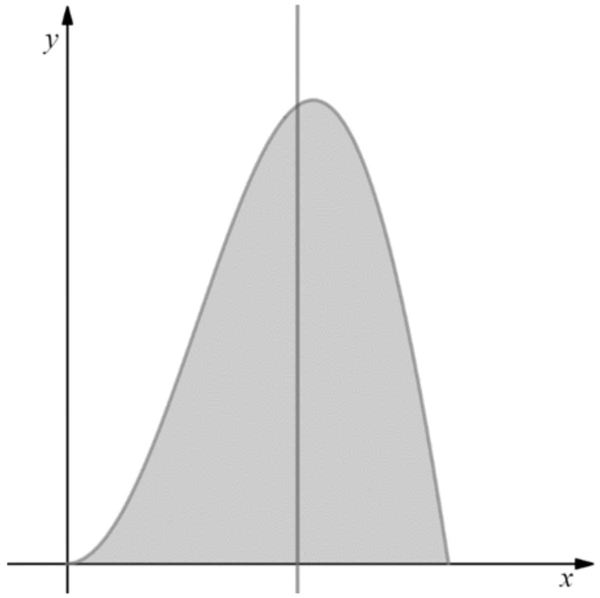
a. Differentiate $x\cos(x)$ 1 mark

b. Hence, show that the area of the shaded region below bounded by the curve with equation $y = 4x\sin(x)$ and the x axis is 4π square units. 4 marks

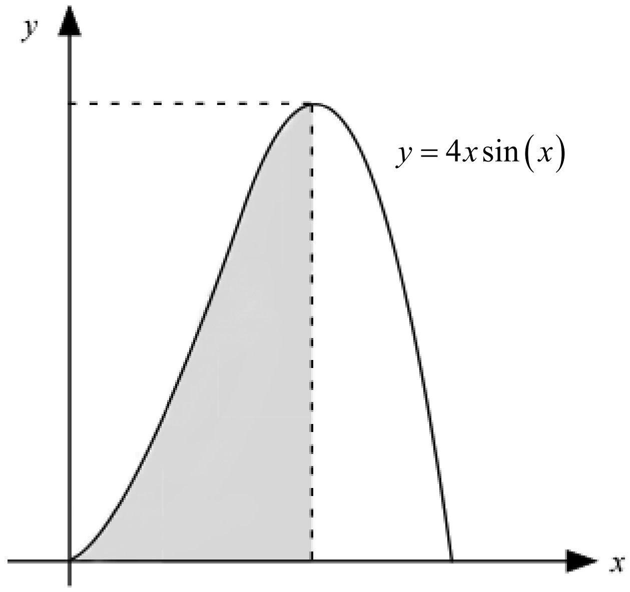


c. The vertical line with equation $x = b$ bisects the area of the same shaded region.

Is b greater than or less than $\frac{\pi}{2}$? Use calculus to justify your response. 3 marks



d. Let k denote the x-coordinate of the turning point. Show that the fraction of the area of the dotted rectangle that is shaded is equal to $\frac{2}{k^2}$. 3 marks



END OF SAC 1b

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$		$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		