	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

Unit 4-SAC 1c – Application Task: Test

Thursday 15th August 2019

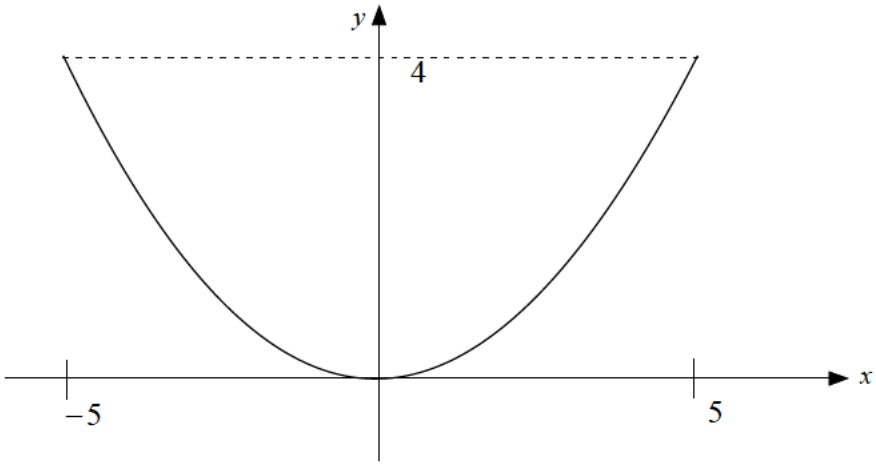
Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	30	

General Instructions
• Answer all questions in the spaces provided. • In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. • In questions where more than one mark is available, appropriate working must be shown. • Unless otherwise indicated, the diagrams in this task are not drawn to scale.
Allowed Materials
• Calculators are allowed • Notes and/or references are not allowed
At the end of the task
• Ensure you cease writing upon request.
Electronic Devices
Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

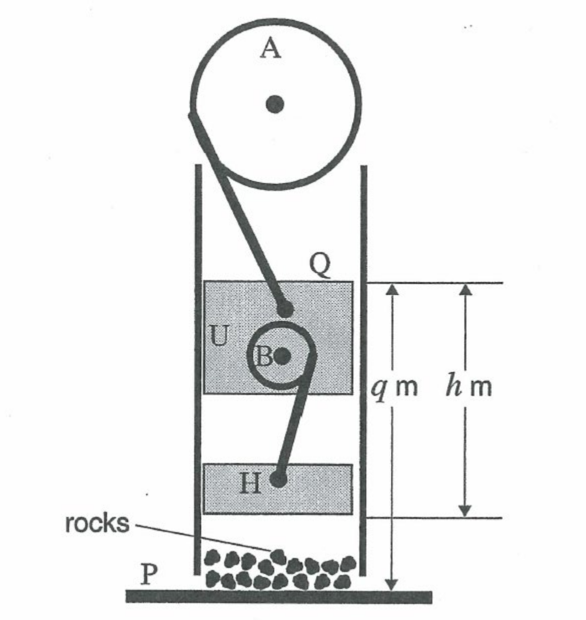
Question 1 (5 marks)

The cross section of a particular waterway is parabolic as shown below. Its depth is 4 metres, and the width across the top of the waterway is 10 metres. When the waterway is 70% full, what is the depth of the water in metres, correct to four decimal places.



Question 2 (11 marks)

A device for crushing rock is shown in the diagram below. It consists of a steel platform (P) on which the rock is placed and a machine which raises and lowers a heavy 'hammer' (H). The wheel A rotates, causing the upper block U to move up and down. The other wheel B, attached to the block U, rotates independently causing the hammer H to move up and down.



Q is the top of block U. The distance, q m, between Q and the platform P is modelled by the formula

$$q(t) = -2\cos(at) + b,$$

where t is the time in minutes and a and b are constants. When $t = 0$, Q is at its lowest point, 3 m above the platform. Wheel A rotates at a rate of 1 revolution per minute.

a. Show that $a = 2\pi$ and $b = 5$. 2 marks

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Wheel B rotates at a rate of 4 revolutions per minute. The distance, h m, between the the bottom of the hammer and Q at time t minutes is modelled by the formula

$$h(t) = -\sin(8\pi t) + 2.$$

Let the distance between the bottom of the hammer and the platform at time t minutes be x m.

b. **i.** Show that $x(t) = -2\cos(2\pi t) + \sin(8\pi t) + 3$. 1 mark

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ii. Write down the period of $x(t)$. 1 mark

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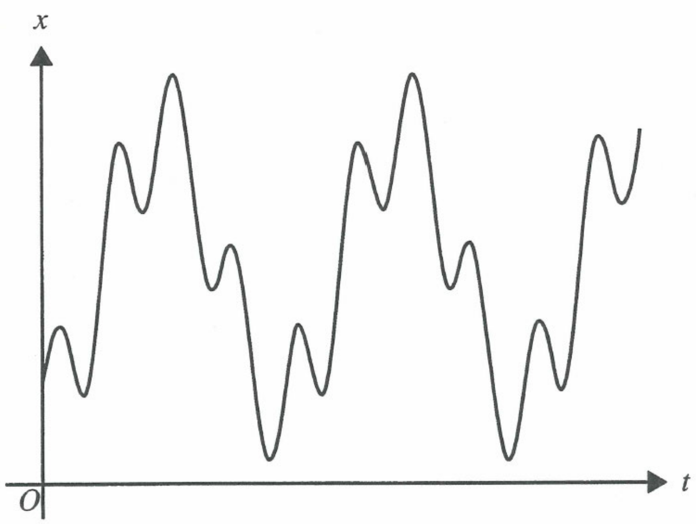
A section of the graph of x as a function of t is shown.



c. Use calculus to find the rate of change of x with respect to t when $t = 2$.
Give your answer correct to one decimal place. 2 marks

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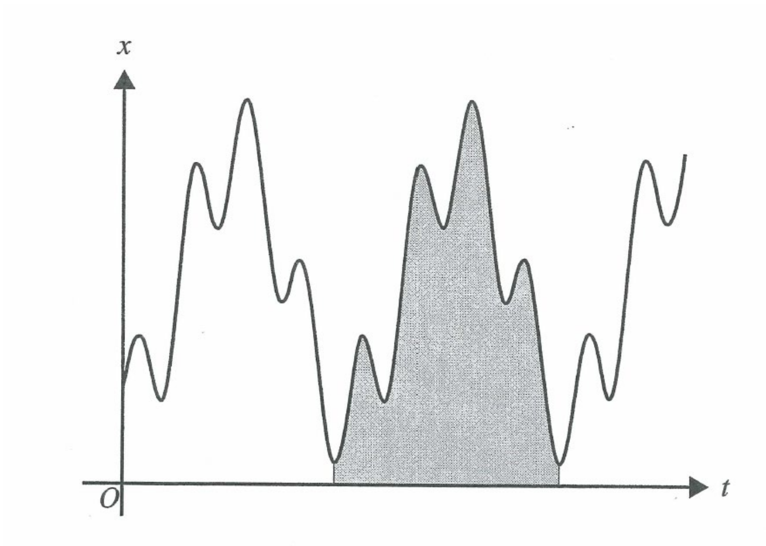
d. Find the first time after $t = 0$, correct to the nearest one-hundredth of a minute, when this model predicts that the bottom of the hammer will be at its least distance from the platform and find this least distance, correct to the nearest *millimetre*. 2 marks

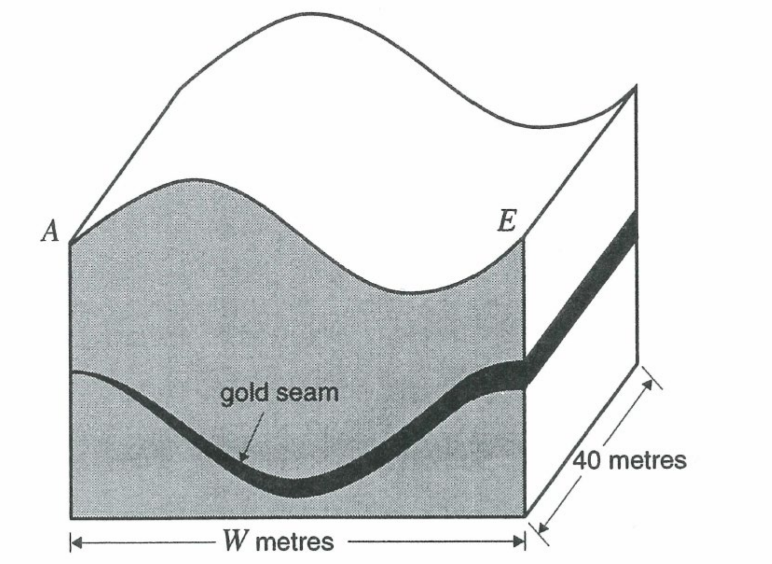
e. The width of the shaded region shown is the time taken for one cycle of x.

Use calculus to find the exact area of the shaded region. 3 marks

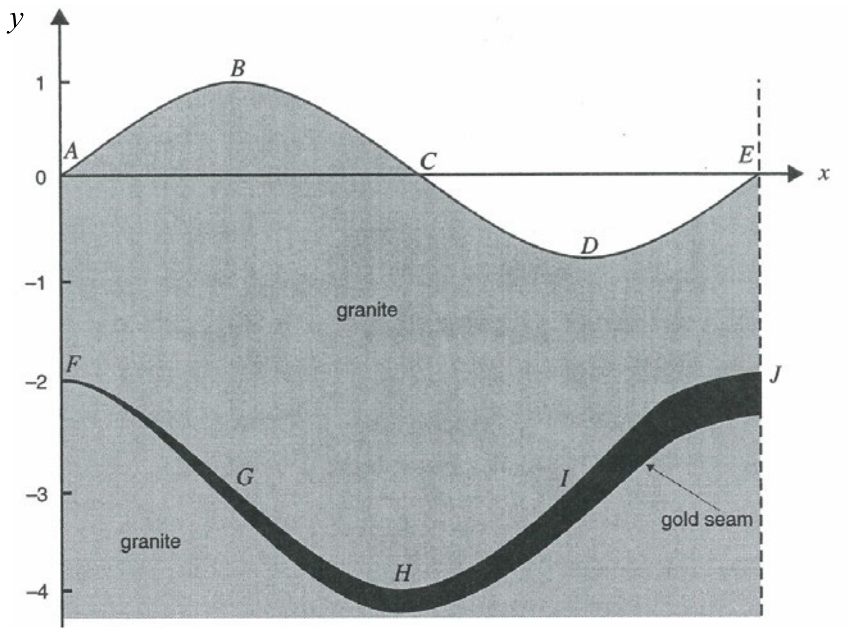


Question 3 (14 marks)

A miner is working at a small mining site shown below. The site is 40 metres long and *W* metres wide. The miner has surveyed the site completely so he knows that it has a constant cross-section all along its 40 metre length. He also knows that a gold seam (layer that contains gold) runs through the site and that it is underneath some granite rock.



The diagram below shows the cross-section of the site and shows the depths and locations of the rock and the gold seam, where *x* is the horizontal distance (in metres) from *A* and *y* is the vertical distance (in metres) above the line *AE*.



The equation $y = \sin\left(\frac{\pi x}{10}\right), \quad 0 \le x \le W$ represents the surface of the rock (*ABCDE*).

The equation $y = \cos\left(\frac{\pi x}{10}\right) - 3, \quad 0 \le x \le W$ represents the top (upper surface) of the gold seam (*FGHIJ*).

a. Show that $W = 20$. 1 mark

b. Write down an expression in terms of x which represents the vertical distance from the surface to the top of the gold seam. 1 mark

c. Use calculus to determine the exact value of the minimum vertical distance from the surface of the mining site to the top of the gold seam. State the exact value for x for which this occurs. (You may assume that the required value of x is in the interval $(0, W)$. You do not need to justify that the distance you find is a minimum) 5 marks

d. The miner decides to remove all the granite above the gold seam.

Determine the exact cross-sectional area of granite that he will remove. 2 marks

e. The vertical thickness of the gold seam is given by

$$T = 0.2 - 0.002(20 - x)^{1.5}, \quad 0 \le x \le W$$

Find the total volume of gold, V m^3, which can be removed from the site, given that the seam contains 0.2% gold by volume.

Give your answer correct to three decimal places. 3 marks

f. The miner decides that it is too expensive to remove all the granite from above the gold seam at his site. He decides to excavate all the granite vertically over the 5 metres between D and E. Calculate the percentage of the total amount of gold which he is now able to mine.

Give your answer correct to the nearest per cent. 2 marks

END OF SAC 2

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) \quad nx^{n \quad 1}$		$\int x^n dx \quad \frac{1}{n+1}x^{n+1}+c,\ n \neq \quad 1$	
$\frac{d}{dx}\left((ax+b)^n\right) \quad an(ax+b)^{n \quad 1}$		$\int (ax+b)^n dx \quad \frac{1}{a(n+1)}(ax+b)^{n+1}+c, n \neq \quad 1$	
$\frac{d}{dx}\left(e^{ax}\right) \quad ae^{ax}$		$\int e^{ax} dx \quad \frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}(\log_e(x)) \quad \frac{1}{x}$		$\int \frac{1}{x}dx \quad \log_e(x)+c,\ x>0$	
$\frac{d}{dx}(\sin(ax)) \quad a\ \cos(ax)$		$\int \sin(ax)dx \quad \frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}(\cos(ax)) = \quad a\ \sin(ax)$		$\int \cos(ax)dx \quad \frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} \quad a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) \quad u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) \quad \frac{v\frac{du}{dx} \quad u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		