	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
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	4	4	4	4
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Teacher's Name

Scotch College

MATHEMATICAL METHODS

Unit 4 - SAC 2a – Application Task: Project

Date of distribution: Tuesday 3rd September 2019

Due date: Friday 13th September 2019

Task Sections	Marks	Your Marks
Extended Response Questions	64	
Total Marks	64	

General Instructions
• Answer all questions in the spaces provided. • In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. • In questions where more than one mark is available, appropriate working must be shown.
Allowed Materials
• A scientific calculator and a CAS calculator. • Any notes or references.
At the end of the task
• Submit the task to your teacher by the due date.

Question 1 (5 marks)

The probability density function of the continuous random variable X is given by

$f(x) = kx^2(1-x), 0 \le x \le 1$ and 0 elsewhere, where k is a real constant.

a. Show that $k = 12$ 2 marks

b. Find exactly the mode of X 3 marks

Question 2 (4 marks)

The continuous random variable, X, has a probability density function given by

$$f(x) = \begin{cases} \frac{2}{3}\left(\sin\left(\frac{x}{3}\right) - \cos\left(\frac{x}{6}\right)\right) & \pi \le x \le 3\pi \\ 0 & \text{elsewhere} \end{cases}$$

a. Find $\Pr\left(X > \frac{13\pi}{6}\right)$, correct to three decimal places. 2 marks

b. Find the value of a, correct to three decimal places, such that $\Pr(X < a) = 4 - 2\sqrt{3}$ 2 marks

Question 3 (17 marks)

A company supplies jewellery stores with watch batteries. The total time for which a battery can be used in a watch is called its life. There are two types of batteries: gold and copper. The life of a gold battery is normally distributed with a mean of 50 weeks and a standard deviation of 3 weeks.

a. Find the probability that a gold battery will have a life that is greater than 51 weeks, correct to three decimal places. 1 mark

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The life of copper batteries is described by the probability density function

$$f(x) = \begin{cases} \dfrac{x}{625}(52-x)\left(e^{\frac{x}{52}}-1\right) & 48 \le x \le 52 \\ 0 & \text{elsewhere} \end{cases}$$

where x is the life in weeks.

b. Determine the expected life of a copper battery. Give your answer in weeks, correct to two decimal places. 2 marks

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c. Determine the standard deviation of the life of a copper battery. Give your answer in weeks, correct to two decimal places. 2 marks

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d. Find the probability that a randomly chosen copper battery will have a life greater than 51 weeks, correct to two decimal places. 2 marks

A large number of unlabeled batteries were mixed in a storage box. The same number of each battery was stored in the box.

e. i. Find the probability that a randomly selected battery will have a life longer than 51 weeks, correct to three decimal places. 2 marks

ii. Find the probability that a gold battery was selected, given that the randomly selected battery has a life longer than 51 weeks. Give your answer correct to three decimal places. 2 marks

When batteries are stored for too long before use, they may no longer give any power, i.e. they will be defective. The company has batteries which are three years old, and, at that age, it is known that 4% of gold batteries will be defective.

f. A jewellery store has a box of 80 gold batteries that are three years old. Find the probability, correct to four decimal places, that at least one of these batteries will be defective. 2 marks

g. Let $\hat{P}$ be the random variable of the distribution of sample proportions of defective gold batteries in boxes of 80. The boxes come from the stock that is three years old.
Find $\Pr\left(\hat{P} > 0.035 \middle| \hat{P} < 0.065\right)$. Give your answer correct to three decimal places.
Do not use a normal approximation. 3 marks

h. A crate of 144 gold batteries that is three years old is inspected and it is found that 10 of the batteries are defective. Determine an 80% confidence interval for the population proportion from this sample, correct to three decimal places. 1 mark

Question 4 (11 marks)

For a particular airline, records show that the probability that a passenger purchases one or more duty free items is 0.55.

a. Pam is an airline steward, and on a particular day she works on a flight with 238 passengers. Let the random variable X represent the number of passengers on the flight who purchase one or more items of duty free goods.

i. Find the probability that the first three customers Pam serves **do not** purchase any duty free items. Give your answer correct to three decimal places. 1 mark

ii. Find the mean number of passengers on the flight who purchase duty free items. 1 mark

iii. Find the probability, correct to 4 decimal places, that at least 130 of the passengers on the flight purchase at least one duty free item. Give your answer correct to three decimal places. 1 mark

b. Find the number of passengers that would need to be on a flight so that the probability of at least one passenger purchasing a duty free item is at least 85%. 2 marks

The airport duty free store has a special on giant Toblerones, and has to frequently re-stock the shelves. The amount of times, Q, which the giant Toblerones sell out and have to be restocked in any given day is a random variable with a distribution given by

Q	0	1	2	3	4
$\Pr(Q = q)$	0.15	0.25	0.3	0.2	0.1

The giant Toblerones had to be restocked on both Wednesday and Thursday.

c. Find the probability that the Toblerones had to be restocked a total of six times over these two days. Give your answer correct to four decimal places. 3 marks

d. Airport Quality Control decides to inspect the giant Toblerones, which are labeled as 600g. It is found that 10% weigh under 596g and 15% weigh over 605g. Based on this information, find the mean and standard deviation for the normal distribution that applies to the mass of the giant Toblerones. Give your answers correct to two decimal places. 3 marks

Question 5 (11 marks)

David works as a quality controller at a factory that produces basketballs. It is his job to decide whether the machines in use need to be reset. Machines are reset when an unacceptable number of defective basketballs are being packaged. On a particular day, two machines are being used to produce basketballs and at the end of the day David is given the following production data:

	Total number of basketballs produced	Number of defective basketballs produced	Number of acceptable basketballs produced
Machine A	640	80	560
Machine B	360	27	333

a. **i.** A basketball produced on this day by machine A is selected at random. What is the probability that it is defective? 1 mark

ii. A sample of 120 basketballs is selected at random from those produced on this day by machine A. What would be the expected number of defective basketballs in this sample? 1 mark

iii. A basketball is selected at random from all the basketballs produced on this day and found to be acceptable. What is the probability, correct to three decimal places, that this basketball was produced by machine A? 2 marks

b. Based on the figures in the above table, find the 95% confidence interval, correct to three decimal places, for the proportion of defective basketballs that machine B produces. 1 mark

As each basketball is produced by machine A or B, it is rolled onto a central conveyor belt. Basketballs from this belt are then packaged in boxes of 6.

c **i.** On this day, David selects a basketball at random from the conveyor belt. What is the probability, correct to three decimal places, that this basketball is defective? 1 mark

ii. What is the probability, correct to three decimal places, that a box of 6 basketballs produced on this day contains no defective basketballs? 2 marks

iii. What is the probability, correct to three decimal places, that a box of 6 basketballs produced on this day contains more than one defective basketball? 2 marks

iv. David has decided that if 5% or more of the boxes contain more than 1 defective basketball, the machines need to be reset before the next day's production. Will the machines need to be reset? Justify your answer. 1 mark

Question 6 (16 marks)

Uncle Albert Jones keeps chickens in his backyard. He regularly records the weights of the eggs they lay and finds that the weights are normally distributed with a mean of 61 grams and a standard deviation of 8 grams.

a. One afternoon, Uncle Albert checks to find a freshly laid egg in the chicken coop. Calculate the probability, correct to four decimal places, that the egg weighs more than 67 grams. 2 marks

b. The next morning, Albert finds 6 freshly laid eggs in the coop. Hence or otherwise, find the the probability, correct to four decimal places, that at least two of these eggs weigh more than 67 grams. 2 marks

Uncle Albert's neighbour, Victor Cab, also keeps chickens which lay eggs whose weights are normally distributed with a variance of only 4 grams.

c. Victor claims that 98% of his eggs weigh more than 67 grams. Show that the mean weight of Victor's eggs can be calculated by the equation $-4.1075+\mu=67$ and hence show that the mean weight of Victor's eggs is 71.1 g, correct to one decimal place. 3 marks

d. 10% of Victor's eggs are less than a mass, m grams. Find the value of m correct to two decimal places. 2 marks

Uncle Albert decides to sneak over to Victor's coop and see the results for himself. He finds that of the 12 eggs that Victor's chickens have laid that day, 7 of them weigh more than 67 grams.

e. Uncle Albert uses this evidence to calculate a 95% confidence interval for the true value of the proportion of Victor's eggs weighing more than 67 grams. Rounded correct to two decimal places, what answer should he get? 1 mark

f. What is Uncle Albert's margin of error, correct to two decimal places? 1 mark

Uncle Albert seeks advice from his local Poultry Hub. He is given the advice that to improve the egg-laying capability of his chickens, the feed intake of his hens needs to be a steady 100 – 110 grams per day. The amount of daily feed consumed by each of Uncle Albert's hens can be approximately modelled by the following continuous probability density function, where y represents the amount of feed eaten by a chicken.

$$f(y)=\begin{cases}\dfrac{3}{\sqrt{y^2-90}} & 90\le y\le n\\ 0 & \text{elsewhere}\end{cases}$$

g. Write an equation that could be used to show that $n=125$, rounded correct to the nearest gram. 1 mark

h. Using $n = 125$, find the expected amount of daily feed for each of Uncle Albert's chickens.

Write your answer correct to two decimal places. 2 marks

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i. Find the probability, correct to two decimal places, that any one of Uncle Albert's hens has a daily feed between 100 and 110 grams. 2 marks

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END OF SAC 2A

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) \quad nx^{n \quad 1}$		$\int x^n dx \quad \frac{1}{n+1}x^{n+1}+c, \; n \neq \quad 1$	
$\frac{d}{dx}\left((ax+b)^n\right) \quad an(ax+b)^{n \quad 1}$		$\int (ax+b)^n dx \quad \frac{1}{a(n+1)}(ax+b)^{n+1}+c, n \neq \quad 1$	
$\frac{d}{dx}\left(e^{ax}\right) \quad ae^{ax}$		$\int e^{ax} dx \quad \frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right) \quad \frac{1}{x}$		$\int \frac{1}{x} dx \quad \log_e(x)+c, \; x>0$	
$\frac{d}{dx}\left(\sin(ax)\right) \quad a \; \cos(ax)$		$\int \sin(ax) dx \quad \frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)= \quad a \; \sin(ax)$		$\int \cos(ax) dx \quad \frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)} \quad a \; \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) \quad u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) \quad \frac{v\frac{du}{dx} \quad u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x - \mu)^2\, p(x)$
continuous	$\Pr(a < X < b) \quad \int_a^b f(x)dx$	$\mu \quad \int_{\infty}^{\infty} x\, f(x)dx$	$\sigma^2 \quad \int_{\infty}^{\infty} (x \quad \mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET