	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
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	7	7	7	7
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	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

U3-SAC 1a – Application Task: Project

Date of distribution: Thursday 16th July 2020

Due date: Tuesday 28th July 2020

Task Sections	Marks	Your Marks
Extended Response Questions	70	
Total Marks	70	

Remote Declaration
I declare that any work I have submitted for this Unit 1 or 3 assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- A scientific calculator and a CAS calculator.
- Any notes or references.

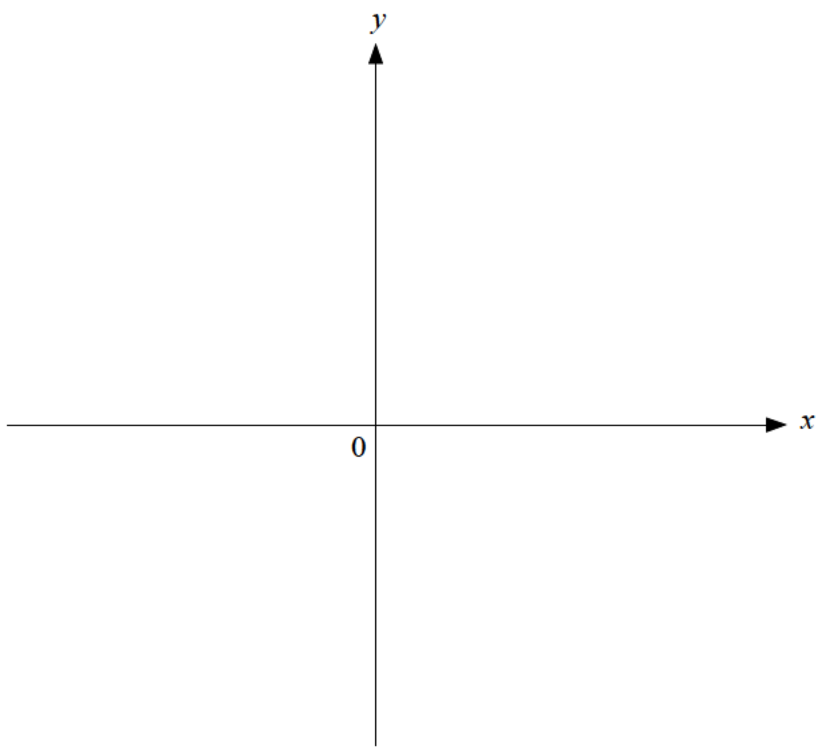
At the end of the task

- Submit the task to your teacher by the due date.

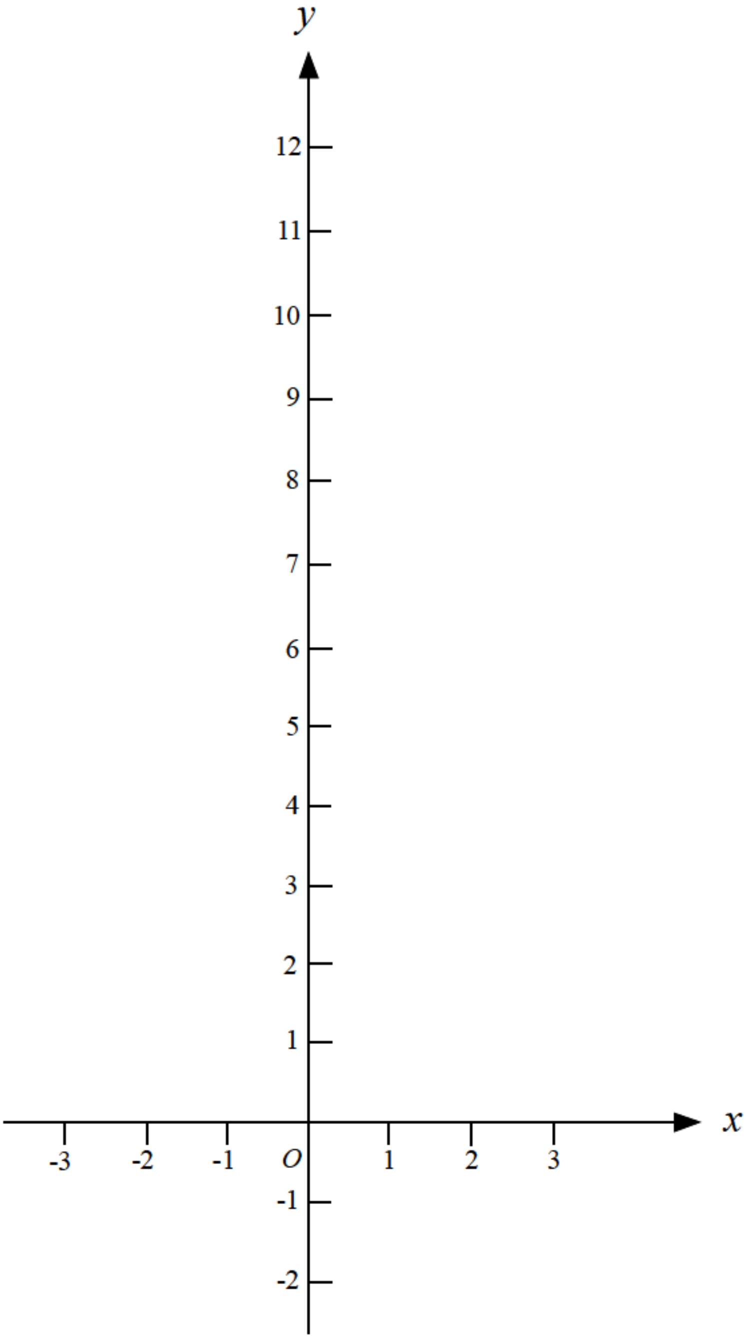
Question 1 (7 marks)

Sketch the graphs for each of the following labelling, with coordinates, stationary points and intersections with the axes where they exist.

a. $y = x^3 - 6x^2 + 12x - 7$ 3 marks



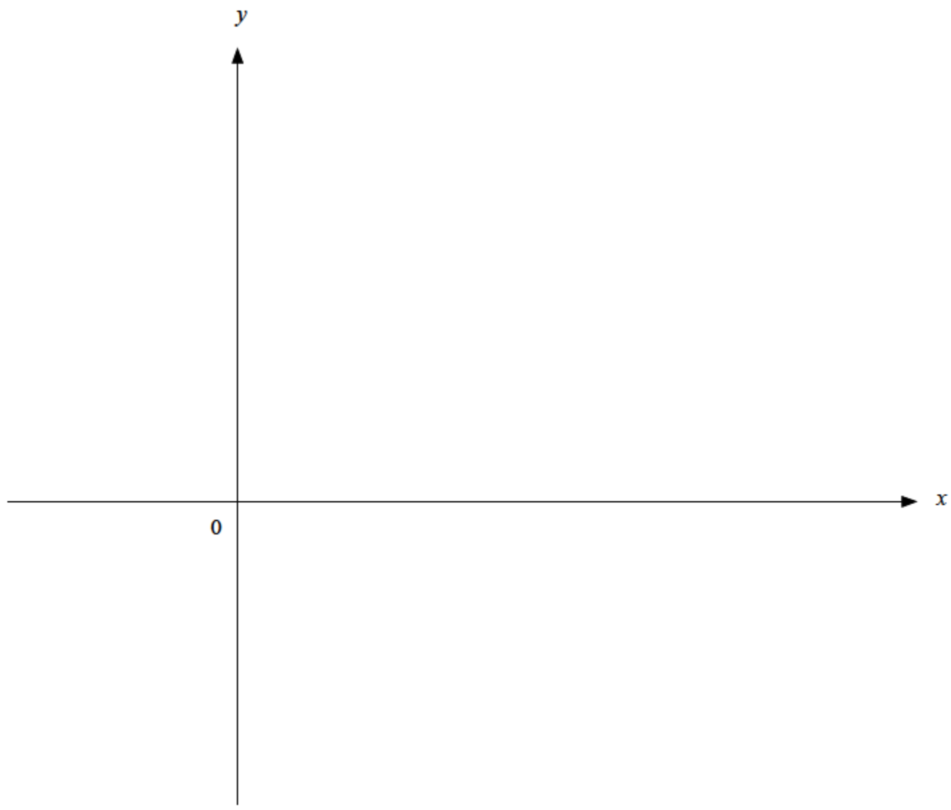
b. $y = x^3 - 3x^2 + 2x + 6, \qquad x \in [-1, 3]$ 4 marks



Question 2 (7 marks)

a. A cubic, in the form $f(x)=ax^3+bx^2+cx+d$, has turning points at $(1,2)$ and $(3,-2)$, determine the equation of the cubic. 3 marks

b. Sketch the graph of $y = f(x)$, labelling axes intercepts and turning points with their coordinates. 3 marks



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c. Find values of q so that the graphs of $y = f(x) + q$ have exactly two x-intercepts. 1 mark

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Question 3 (5 marks)

Given $g(x) = x^3 + kx^2 + 9x - 2$, determine the value(s) of k so that the graph of $y = g(x)$ has

a. one stationary point 3 marks

b. two stationary points 1 mark

c. no stationary points 1 mark

Question 4 (8 marks)

Let $h(x) = x^3 + mx^2 + nx \quad ,(m, n \neq 0)$

a. Determine the conditions for *m* and *n* so that the graph of $y = h(x)$ has two stationary points.

Indicate values of *m* in terms of *n* for the cases $n < 0$ and $n > 0$. 4 marks

b. Hence determine, for the cubic $h(x)$ which has two stationary points, the conditions for *m* and *n* so that the cubic also has only one *x*-intercept. Indicate values of *m* in terms of *n* for the cases $n < 0$ and $n > 0$. 4 marks

Question 5 (14 marks)

a. Consider the function $f:[0,1]\rightarrow \mathbb{R}, f(x)=x(x+6)$. The graph of $y=g(x)$ is the graph of $y=f(x)$ reflected in the line $y=x$.

i. Find the rule for $y=g(x)$. 3 marks

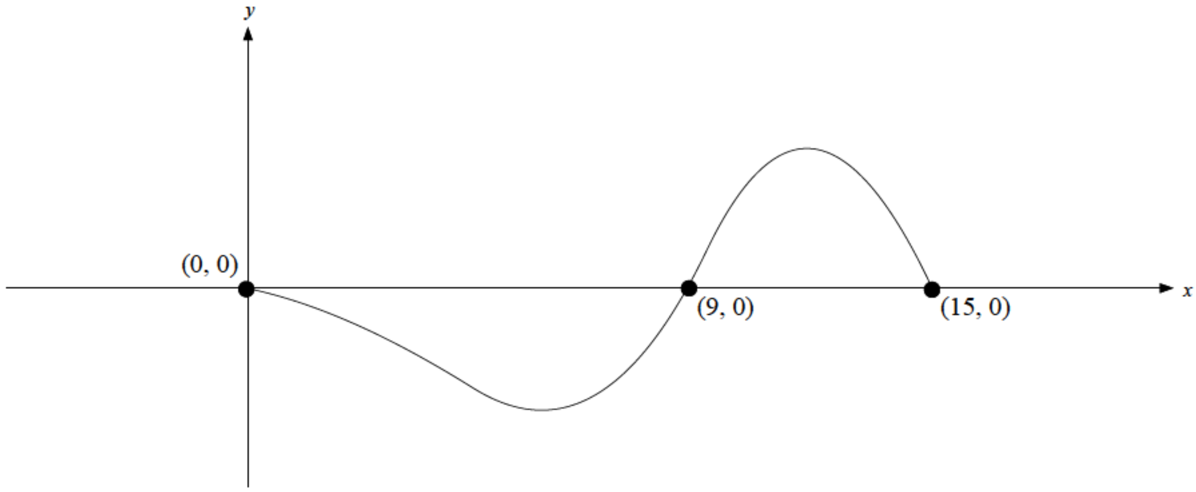
ii. State the domain for $g(x)$. 1 mark

b. List the sequence of transformations which map the graph of $y=\frac{1}{x}$ to the graph of $y=\frac{-A}{x-b}-k$ where A, b and $k\in\mathbb{Z}^{+}$. 4 marks

c. The graph of $y = h(x)$ joins smoothly to the graph of $y = g(x)$ at the point where $x = 7$.

Graphs join smoothly when they have the same gradient at the point of joining. Given $h(15) = 3$ and $h(x) = \frac{-A}{x-b} - k$, find the rule for $h(x)$. 6 marks

Question 6 (7 marks)



The graph drawn above is defined by

$$f(x) = \begin{cases} g(x), & 0 \le x \le 9 \\ h(x), & 9 < x \le 15 \end{cases}$$

$g(x)$ is a cubic and $h(x)$ is a quadratic.

a. For the graph $y = g(x)$, $g(9) = 0$, $g(1) = \dfrac{-2}{3}$ and at $(0,0)$ the gradient is zero. Determine the rule for $g(x)$. 3 marks

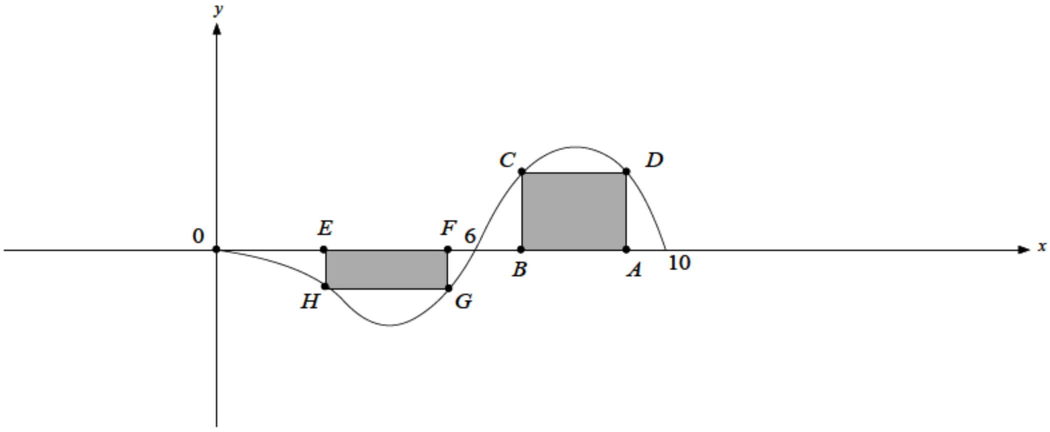
b. For the graph $y = h(x)$, $h(15) = 0$ and it joins smoothly with the graph $y = g(x)$ at $(9, 0)$.

Determine the rule for $h(x)$. 4 marks

Question 7 (13 marks)

Let

$$f(x)=\begin{cases}\frac{1}{3}x^3-2x^2, & 0\le x\le 6\\ -3x^2+48x-180, & 6<x\le 10\end{cases}$$



a. A rectangle $ABCD$ is to be drawn with A and B on the x-axis and C and D on the parabola.

Let $B=(x,0)$.

i. Find an expression $A_1(x)$, in terms of x, for the area of rectangle $ABCD$. 2 marks

ii. Use calculus to determine the dimensions of the rectangle when its area is a maximum and find the area. 5 marks

b. A rectangle $EFGH$ is to be drawn with E and F on the x-axis and G and H on the cubic.

Let $E=(x,0)$ and $F=(x+h,0)$, where $h>0$.

i. Find an expression $A_2(x)$, in terms of x, for the area of rectangle $EFGH$. 3 marks

ii. Determine the dimensions, correct to two decimal places, of the rectangle when its area is a maximum. 3 marks

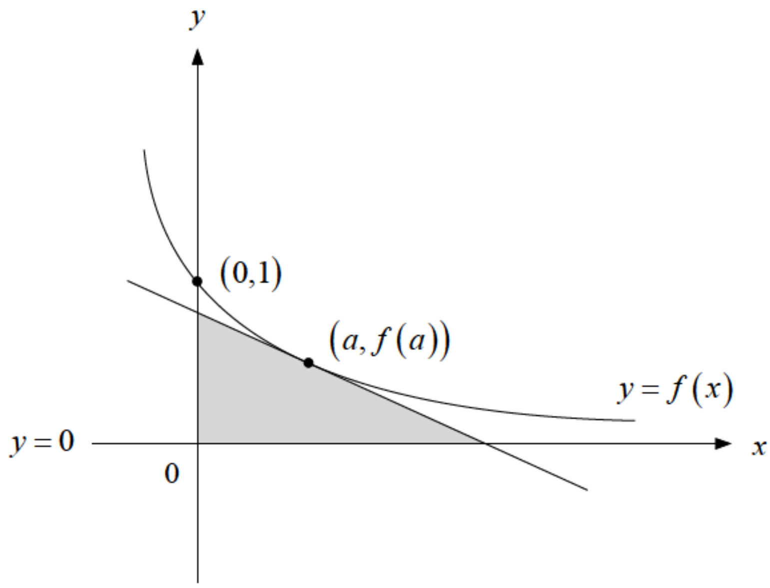
Question 8 (9 marks)

Let $f:\mathbb{R}\rightarrow\mathbb{R}, f(x)=e^{-x}$.

a. **i.** Find the equation of the tangent to the graph of f at the point $(0, 1)$. 2 marks

ii. Find the area of the triangular region bounded by the tangent found in **part a i,** the positive x-axis and the positive y-axis. 1 mark

b. A tangent to the graph of f is drawn at the point $(a, f(a))$, where $a > 0$, to form a triangular region bounded by the tangent, the positive x-axis and the positive y-axis. Use calculus to find the value of a when the area of a triangle is a maximum. 6 marks



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