	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

Unit 3-SAC 1b – Application Task: Test

Tuesday 28th July 2020

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	30	

General Instructions
• Answer all questions in the spaces provided. • In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. • In questions where more than one mark is available, appropriate working must be shown. • Unless otherwise indicated, the diagrams in this task are not drawn to scale.
Allowed Materials
• Calculators are not allowed • Notes and/or references are not allowed.
At the end of the task
• Ensure you cease writing upon request.
Electronic Devices
Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (13 marks)

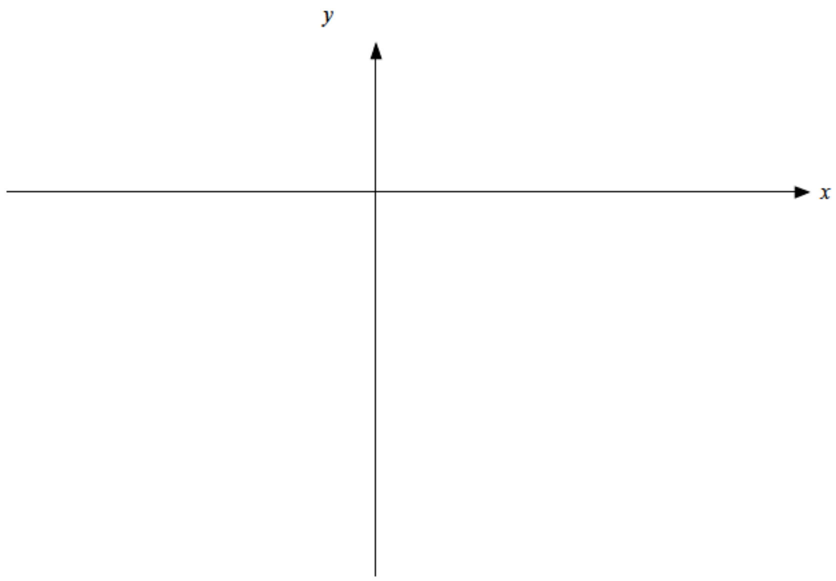
a. Let $f(x) = x^3 + 9x^2 + 15x - 25$.

i. Show that $x - 1$ is a factor of $f(x)$. 1 mark

ii. Hence, solve $f(x) = 0$. 2 marks

iii. Find the coordinates of the stationary points for the graph of $y = f(x)$. 3 marks

iv. Sketch the graph of $y = f(x)$ labelling the axis intercepts and stationary points with their coordinates. 2 marks



b. Let $g(x) = x^3 + 9x^2 + mx + k$.

i. Find the value of m so that the graph of $y = g(x)$ has exactly one stationary point. 3 marks

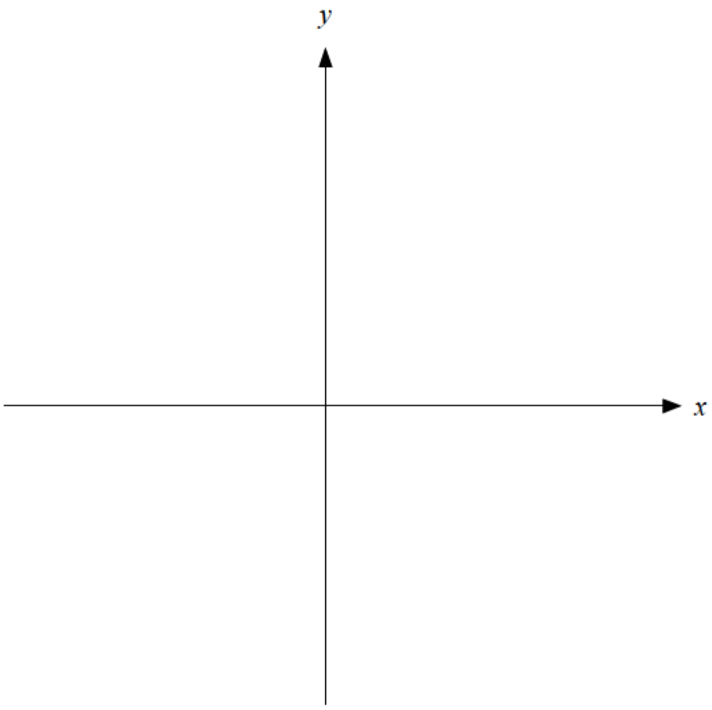
ii. Hence, find the value of k so that the stationary point found in **part i** is on the x-axis. 2 marks

Question 2 (8 marks)

Let $f(x) = 2\log_e(x-1)$.

a. State the maximal domain for the function f. 1 mark

b. Sketch the graph of $y = f(x)$. Label the intercept(s) with coordinates and the asymptote(s) with equations. 2 marks



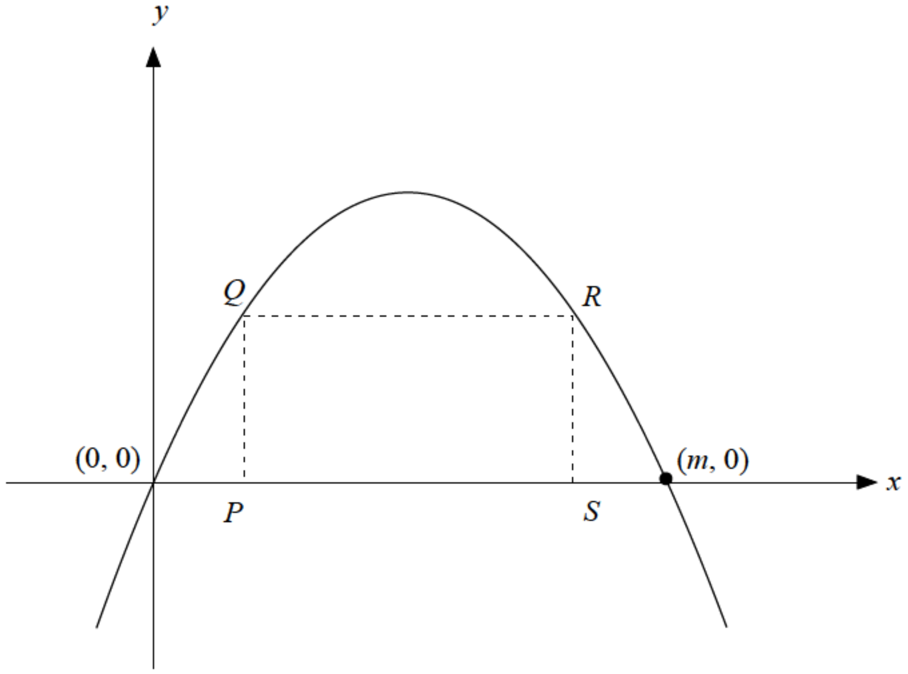
c. **i.** Find the rule for f^{-1}. 2 marks

ii. State the domain for f^{-1}. 1 mark

d. State the sequence transformations which map the graph of $y = \log_e(x)$ to the graph of $y = 2\log_e(x-1)$. 2 marks

Question 3 (9 marks)

The parabola drawn has equation $y = -2x(x-m)$ where $m > 0$.



A rectangle *PQRS*, is drawn so that:

- *P* and *S* are on the *x*-axis
- *Q* and *R* are on the parabola

as shown in the diagram.

Let $P = (x, 0)$.

a. **i.** Show that the rule of the function $A(x)$ for the area of the rectangle *PQRS* is

$$A(x) = 4x^3 - 6mx^2 + 2m^2x$$

2 marks

ii. State the domain for the function A in terms of m. 1 mark

b. **i.** Show that the area of the rectangle *PQRS* is a maximum when $x = \frac{m}{2} - \frac{m\sqrt{3}}{6}$. 3 marks

ii. Find *PS* in the form $\frac{\sqrt{a}}{b}m,$ where $a, b \in \mathbb{Z}$. 1 mark

iii. Find *PQ* in the form $\frac{m^2}{c}$ where $c \in \mathbb{Z}$. 2 marks

END OF SAC 1b

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) \quad nx^{n \quad 1}$	$\int x^n dx \quad \frac{1}{n+1}x^{n+1}+c,\ n \neq \quad 1$
$\frac{d}{dx}\left((ax+b)^n\right) \quad an(ax+b)^{n \quad 1}$	$\int (ax+b)^n dx \quad \frac{1}{a(n+1)}(ax+b)^{n+1}+c, n \neq \quad 1$
$\frac{d}{dx}\left(e^{ax}\right) \quad ae^{ax}$	$\int e^{ax}dx \quad \frac{1}{a}e^{ax}+c$
$\frac{d}{dx}(\log_e(x)) \quad \frac{1}{x}$	$\int \frac{1}{x}dx \quad \log_e(x)+c,\ x>0$
$\frac{d}{dx}(\sin(ax)) \quad a\ \cos(ax)$	$\int \sin(ax)dx \quad \frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}(\cos(ax)) = \quad a\ \sin(ax)$	$\int \cos(ax)dx \quad \frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} \quad a\ \sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) \quad u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) \quad \frac{v\frac{du}{dx} \quad u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x - \mu)^2\, p(x)$
continuous	$\Pr(a < X < b) \quad \int_a^b f(x)dx$	$\mu \quad \int_{\infty}^{\infty} x\, f(x)dx$	$\sigma^2 \quad \int_{\infty}^{\infty} (x \quad \mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET