	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

Unit 3-SAC 1c – Application Task: Test

Tuesday 28th July 2020

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	30	

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- A scientific calculator and CAS calculator are allowed.
- Notes and/or references are not allowed.

At the end of the task

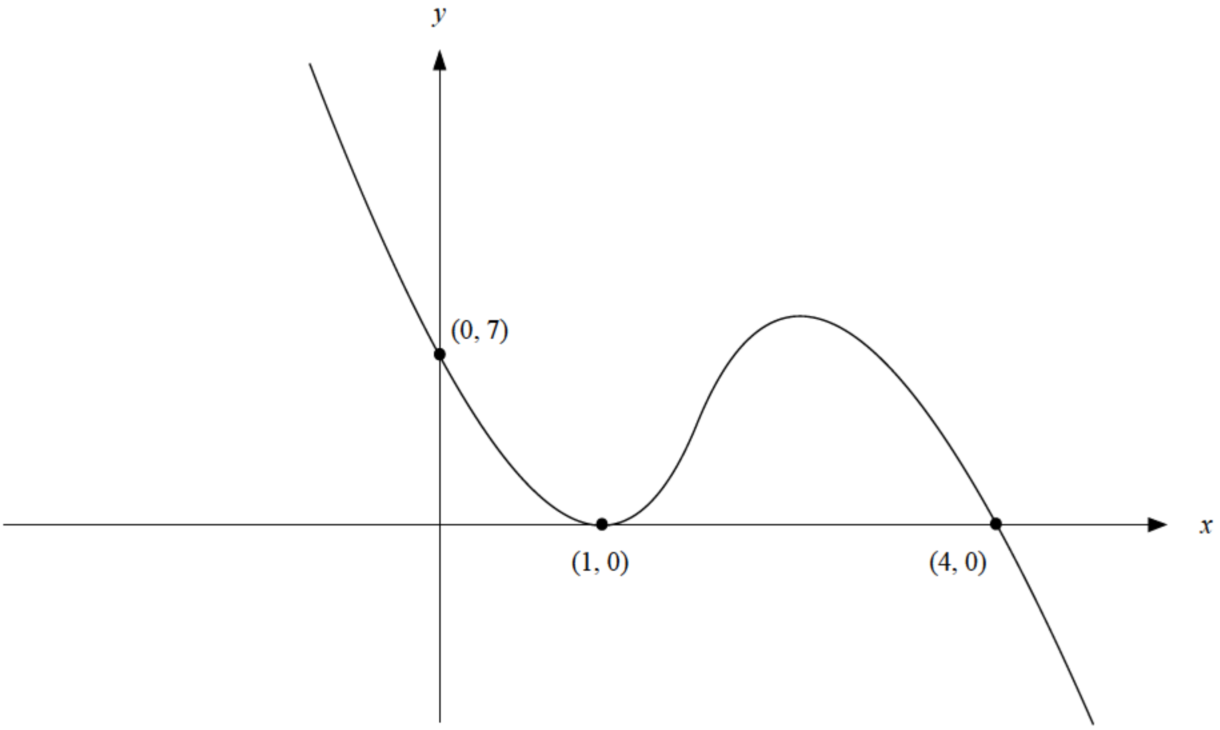
- Ensure you cease writing upon request.

Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

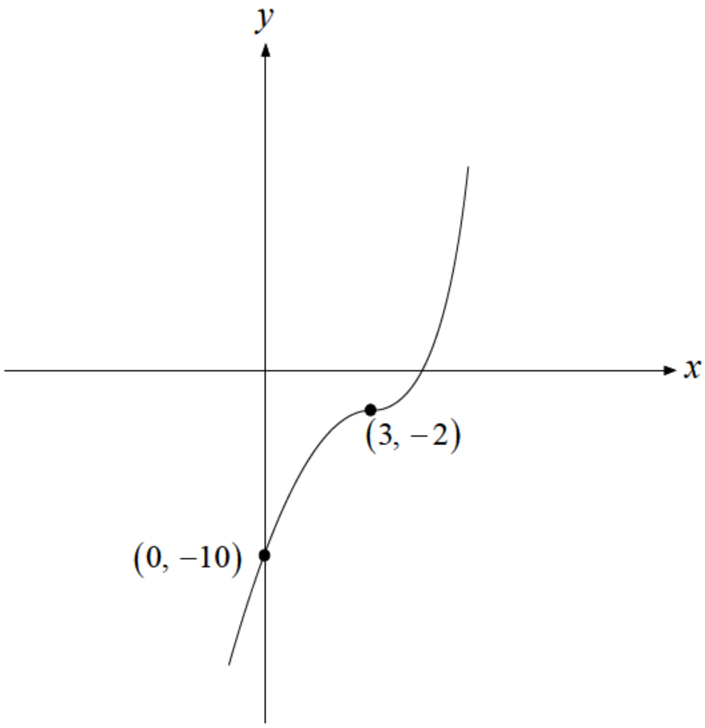
Question 1 (8 marks)

a. $f(x)$ is a polynomial of degree 3. The graph of $f(x)$ is shown below:



Find the rule for $f(x)$ in factorised form. 2 marks

b. $g(x)$ is a polynomial of degree 3, with one stationary point at $(3, -2)$. The graph of $g(x)$ is shown below:



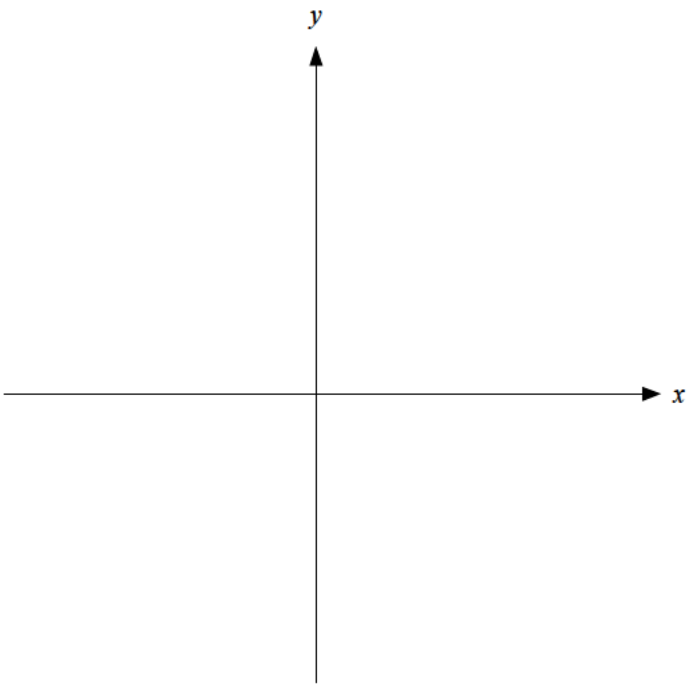
Find the rule for $g(x)$. 3 marks

c. A cubic function $h(x)$ has turning points at $(3,5)$ and $(-2,10)$. Find the rule for $h(x)$ in the form $h(x) = ax^3 + bx^2 + cx + d$. 3 marks

Question 2 (12 marks)

Let $f:\mathbb{R}\to\mathbb{R},\ f(x)=5-(x+2)^2$.

a. Sketch the graph of $y=f(x)$, labelling the turning point and the axes intercepts with their coordinates. 3 marks



b. Find the equation of the tangent to the curve $y=f(x)$ at the point where $x=-3$. 2 marks

c. The tangent to the curve $y = f(x)$ at the point $(b, f(b))$ passes through the point $(1, 0)$.

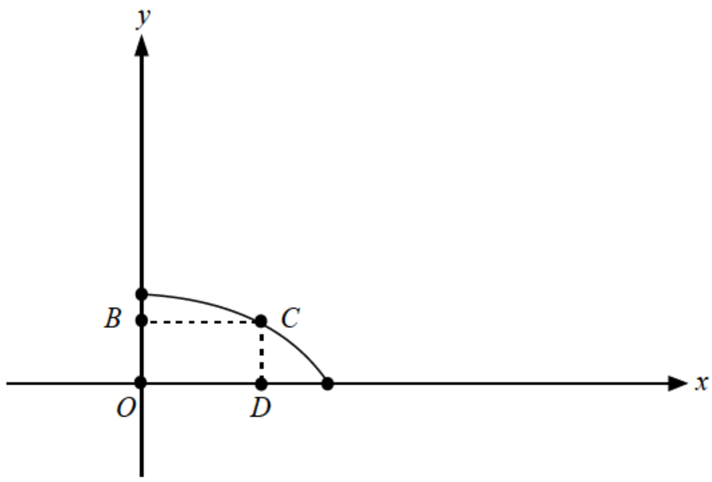
Find the value(s) of b. 3 marks

d. Given $g(x) = a(x-4)^2 - 3, \quad a > 0$ and that the graphs of f and g are to join smoothly (i.e. join at a point where they have the same gradient), find the value of a and the coordinates of the point at which the parabolas join. 4 marks

Question 3 (6 marks)

a. Determine the sequence of transformations which map the graph of $y = xe^x$ to the graph of $y = -(x-1)e^{x-1}$. 2 marks

b. A rectangle is drawn with O at the origin, B on the y-axis, C on the curve with equation $y = (1-x)e^{x-1}$ and D on the x-axis. Let $D = (x, 0)$.



i. Write down the rule for the function, $A(x)$, for the area of the rectangle $OBCD$. 1 mark

ii. Use calculus to find the value of x for the area of rectangle $OBCD$ to be a maximum. 3 marks

Question 4 (4 marks)

Let $f(x) = x^2 + 8x + 6$ and $g(x) = -x^2 + 6x - 7$.

The graphs of $y = f(x)$ and $y = g(x)$ are to be joined by a line which is a tangent to both curves.

Find the equations of all possible lines.

END OF SAC 1c

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) \quad nx^{n \quad 1}$		$\int x^n dx \quad \frac{1}{n+1}x^{n+1}+c,\ n \neq \quad 1$	
$\frac{d}{dx}\left((ax+b)^n\right) \quad an(ax+b)^{n \quad 1}$		$\int (ax+b)^n dx \quad \frac{1}{a(n+1)}(ax+b)^{n+1}+c, n \neq \quad 1$	
$\frac{d}{dx}\left(e^{ax}\right) \quad ae^{ax}$		$\int e^{ax}dx \quad \frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right) \quad \frac{1}{x}$		$\int \frac{1}{x}dx \quad \log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right) \quad a\ \cos(ax)$		$\int \sin(ax)dx \quad \frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)= \quad a\ \sin(ax)$		$\int \cos(ax)dx \quad \frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)} \quad a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) \quad u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) \quad \frac{v\frac{du}{dx} \quad u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) \quad \int_a^b f(x)dx$	$\mu \quad \int_{\infty}^{\infty} x\, f(x)dx$	$\sigma^2 \quad \int_{\infty}^{\infty} (x \quad \mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET