	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
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Teacher's Name

Scotch College

MATHEMATICAL METHODS

U4-SAC 1a – Application Task: Project

Date of distribution: Thursday 27th August 2020

Due date: Thursday 3rd September 2020, prior to SAC 1b

Task Sections	Marks	Your Marks
Extended Response Questions	68	
Total Marks	68	

Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: _____

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- A scientific calculator and a CAS calculator.
- Any notes or references.

At the end of the task

- Submit the task to your teacher by the due date.

Question 1 (18 marks)

Consider the graph of f where

$$f:[-2\pi, 2\pi] \to \mathbb{R},\ f(x) = x\sin(x).$$

a. Find the x-intercepts. 2 marks

b. Find the total area of the regions bounded by graph $y = f(x)$ and x-axis over the following intervals.

i. $[-\pi, \pi]$ 2 marks

ii. $[-2\pi, 2\pi]$ 2 marks

c. In addition, now consider the graph of g where $g:[-2\pi,2\pi]\to\mathbb{R}, g(x)=\sin(x)$.

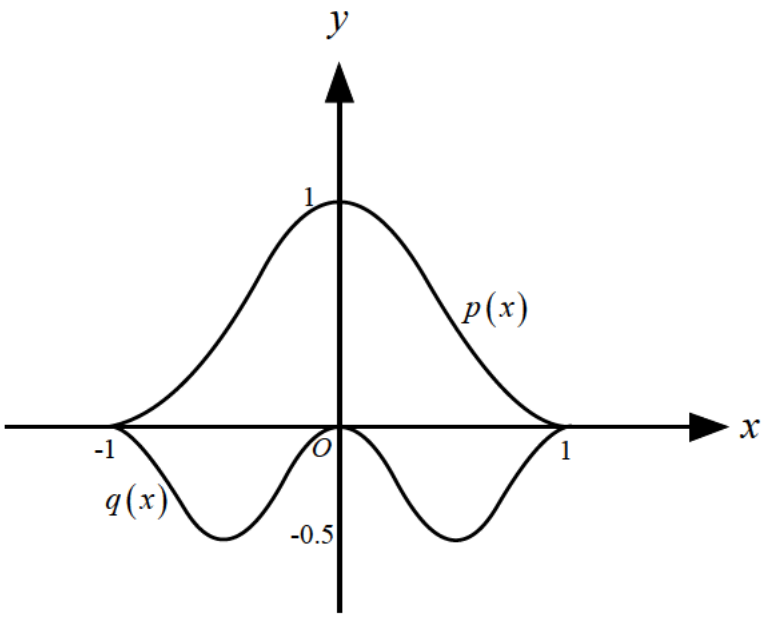
Find the area of the regions enclosed by the graphs of $y=f(x)$ and $y=g(x)$ over each of the intervals given below. In each case write down the integral/s that give the area of the regions and give answers in exact form.

i. $[-\pi,0]$ 2 marks

ii. $[0,\pi]$ 2 marks

iii. $[-2\pi,2\pi]$ 2 marks

d. Consider the two functions p and q as drawn in the diagram. The functions are of the form $p:[-1,1]\to\mathbb{R},\ p(x)=A\cos(nx)+k$ and $q:[-1,1]\to\mathbb{R},\ q(x)=B\cos(mx)+c$.



i. Determine the rules for the functions p and q. 2 marks

ii. Find the area of the region enclosed by the graphs of p and q. 2 marks

iii. The region found in **part d ii** is reduced to become a quarter of its original area by dilations of factor j from the x and y axes. Determine the rules for two new functions p_2 and q_2. 2 marks

Question 2 (16 marks)

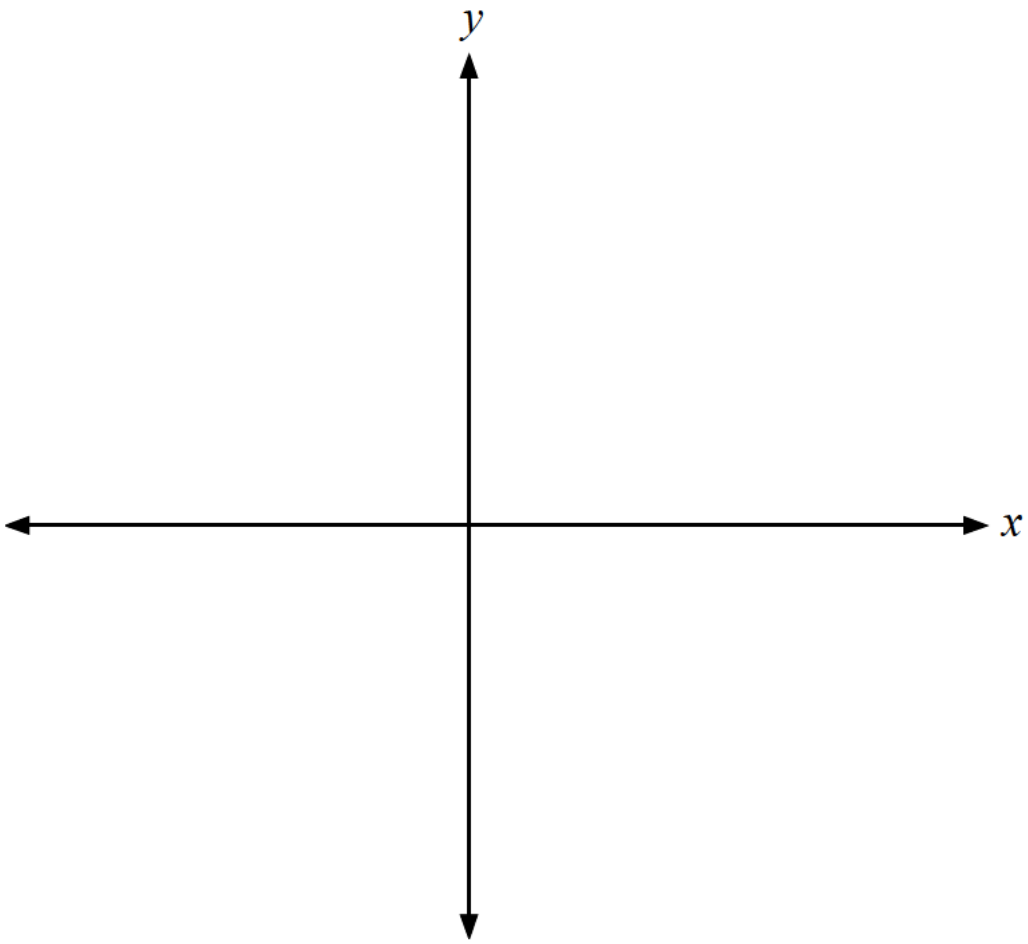
Let f be the function $f : D \to \mathbb{R}, \quad f(x) = \log_e(3-4x)$, where D is the largest possible domain over which f is defined.

a. Find the exact coordinates of the intercepts of the graph of $y = f(x)$ with the x- and y-axes. 2 marks

b. Find D, the largest possible domain over which f is defined. 1 mark

c. Show that the rate of change of $f(x)$ with respect to x is always negative. 2 marks

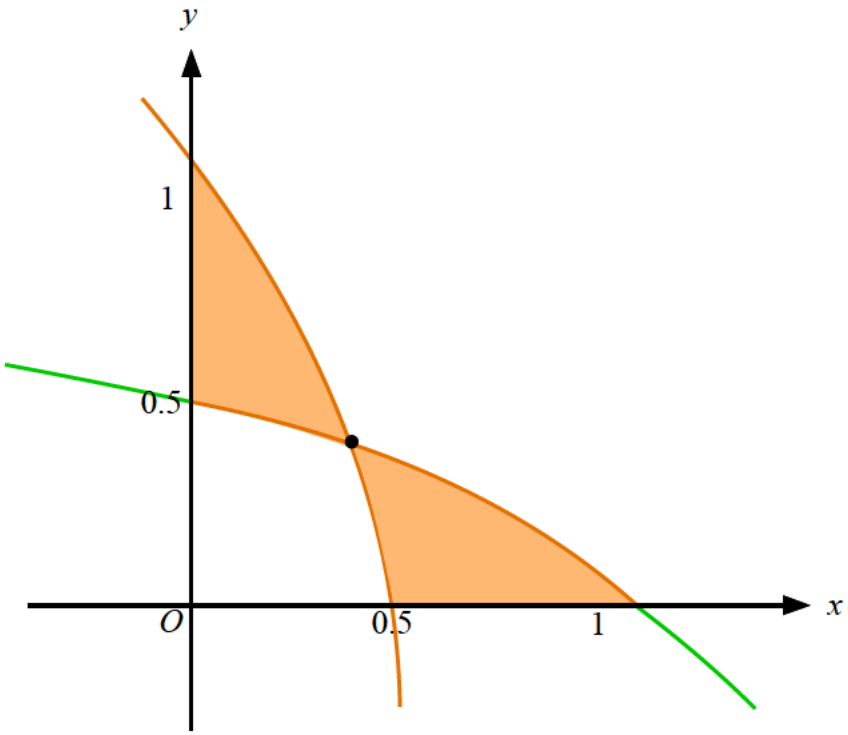
d. On the axes below, sketch the graph of $y = f(x)$. Show clearly the coordinates of any axis intercepts and label any asymptote with its equation. 2 marks



e. f^{-1} is the inverse of f. Find the rule of f^{-1} and state its domain. 3 marks

f. Hence, show that the area bounded by the graph of $y = f(x)$, the x-axis and the y-axis is $\frac{3\log_e 3}{4} - \frac{1}{2}$. 3 marks

On the axes below, parts of the graphs of f and f^{-1} are shown.



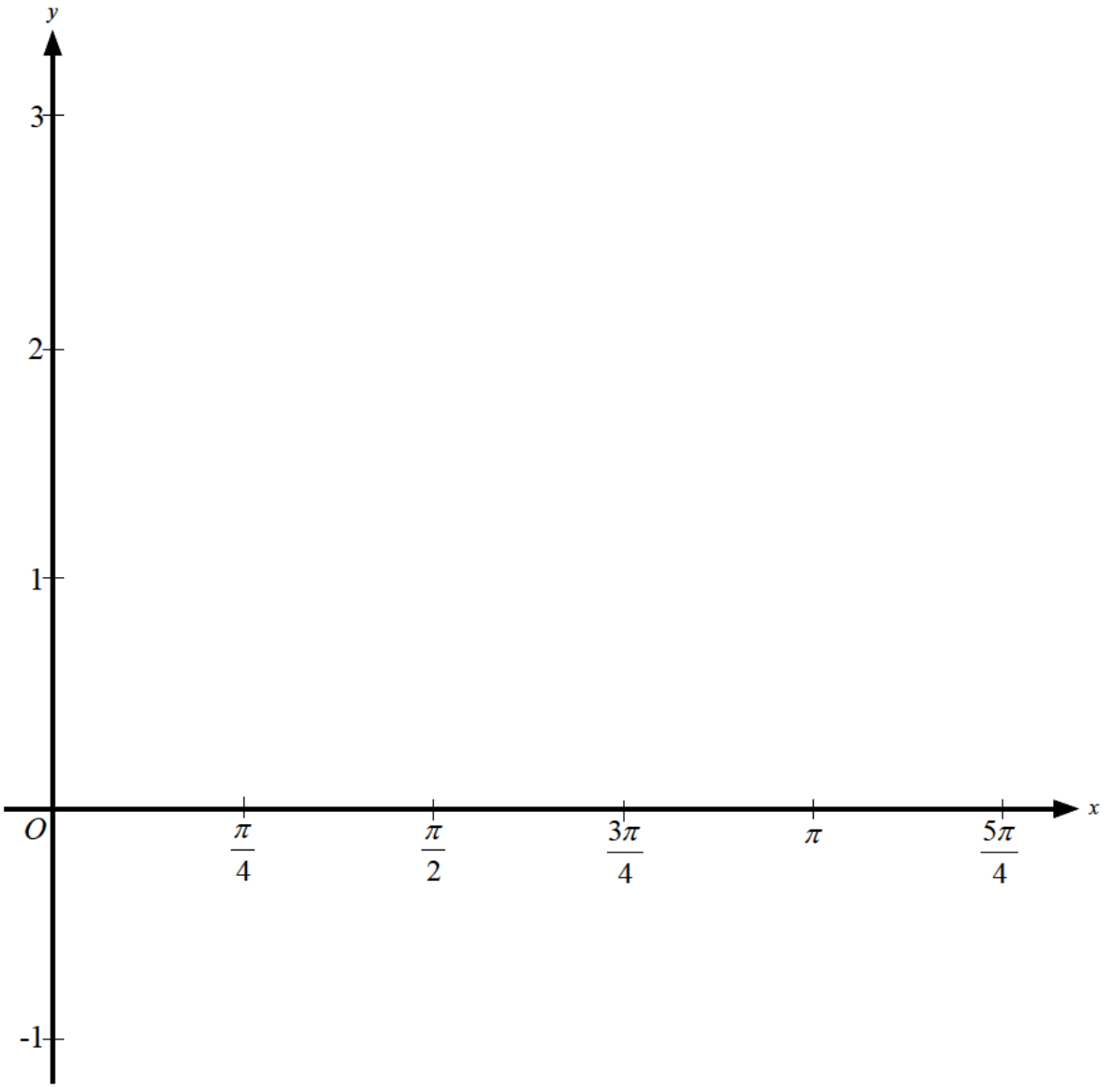
g. Find the area of the shaded region in the above diagram, correct to two decimal places. 3 marks

Question 3 (14 marks)

Consider the two functions f and g

$$f:\left[\frac{\pi}{4},\frac{5\pi}{4}\right]\to\mathbb{R},\quad f(x)=\sin\left[2\left(x-\frac{\pi}{4}\right)\right]+2 \text{ and } g:\left[\frac{\pi}{4},\frac{5\pi}{4}\right]\to\mathbb{R},\quad g(x)=-3\sin\left(x-\frac{\pi}{4}\right)+2.$$

a. On the same set of axes, sketch both functions f and g. Show on your graph the coordinates of the points of intersection and turning points of the two graphs. The x-axis intercepts are not required. 4 marks



b. Find the area of the regions enclosed by the graphs of f and g. 2 marks

c. A line segment is drawn, parallel to the y-axis, joining the graphs of f and g. Let $h(x)$ be the length of the line segment. Find the value of x for which $h(x)$ is a maximum and find the maximum value. Give both answers correct to two decimal places. 4 marks

d. The region enclosed by the graphs of f and g is to be divided into two equal parts by a vertical line (parallel to the y-axis). Find the x-intercept of this line and the coordinates of its points of intersection with the graphs of f and g. Give your answers correct to two decimal places. 4 marks

Question 4 (20 marks)

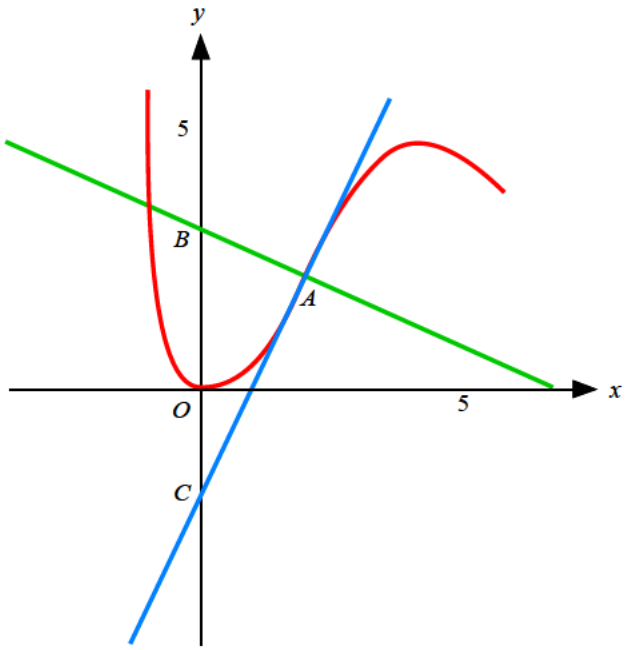
Consider the function f, such that $f:\mathbb{R}\to\mathbb{R}, f(x)=\dfrac{x^4}{e^x}$.

a. Let point A be where $x=2$. State the value of the y-coordinate of this point. 1 mark

b. Find the equation of the tangent to the curve $y=f(x)$ at the point A. 1 mark

c. Find the equation of the normal to the curve $y=f(x)$ at the point A. 1 mark

d. B is the y-intercept of the normal to the curve $y = f(x)$ at A and C is the y-intercept of the tangent to the curve $y = f(x)$ at A. Find the area of triangle ABC and express your answer in the form of $\frac{e^p + q}{re^s}$ where p, q, r and $s \in \mathbb{Z}$. 2 marks



e. Let D be the point on the curve $y = f(x)$ where $x = a$ and $\frac{1}{2} \leq a \leq 3$.

i. Find the equation of the tangent to the curve $y = f(x)$ at point D in the form $y = mx + d$, where m and d are in terms of a. 1 mark

ii. Find the equation of the normal to the curve $y = f(x)$ at point D in the form $y = nx + v$, where n and v are in terms of a. 1 mark

iii. Hence, show that the area bounded by the tangent and normal to the curve $y = f(x)$ at D and the y-axis is given by

$$g: \left[\frac{1}{2}, 3\right] \to \mathbb{R}, \quad g(a) = \frac{e^{2a} + a^6(a-4)^2}{2ae^a(4-a)}$$

3 marks

f. For the region described in **part e**, find the value of a, correct to four decimal places, for the area of the region to be a

i. minimum 1 mark

ii. maximum 1 mark

g. A rectangle is to be drawn with vertices $(0,0),(b,0),(0,c)$ and (b,c) where $b>0$ and $c>0$. c is the average value of $f(x)$ over $[0,b]$. The rectangle is to have an area of 20 square units. Find the values of b and c correct to one decimal place. 3 marks

h. **i.** Find the area of the region between the curve $y = f(x)$ and the x-axis for $x \geq 0$. 2 marks

ii. A line passes through the local maximum at $x = 4$ and a point $(k, 0)$ where $k > 0$.
Find the value of k, correct to four decimal places, so that the line bisects the region between the curve $y = f(x)$ and the x-axis for $x \geq 0$. 3 marks

END OF SAC 1a