	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

Unit 3-SAC 1c – Application Task: Test

Wednesday 2nd June 2021

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	**30**	

Declaration
I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion. *Signature:* ________________________________

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are allowed
- Notes and/or references are allowed.

At the end of the task

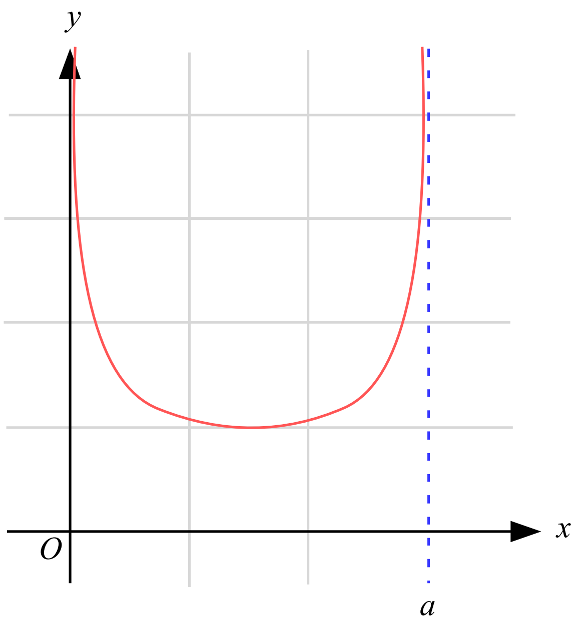
- Ensure you cease writing upon request.

Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (13 marks)

The graph of the function $f:(0,a)\to\mathbb{R}, f(x)=\dfrac{3}{2\sqrt{3x-x^2}}$, including asymptotes at $x=0$ and $x=a$ is shown below.



a. Show that $a=3$. 1 mark

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b. Write down the equations of asymptotes of the curve $y=f\left(\dfrac{x}{2}\right)+3$. 1 mark

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c. Let $g:(0,c]\to\mathbb{R}, g(x)=f(x)$, where c is the largest value of x such that g has an inverse function.

i. State c. 1 mark

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ii. Hence or otherwise, find the absolute minimum of f. 1 mark

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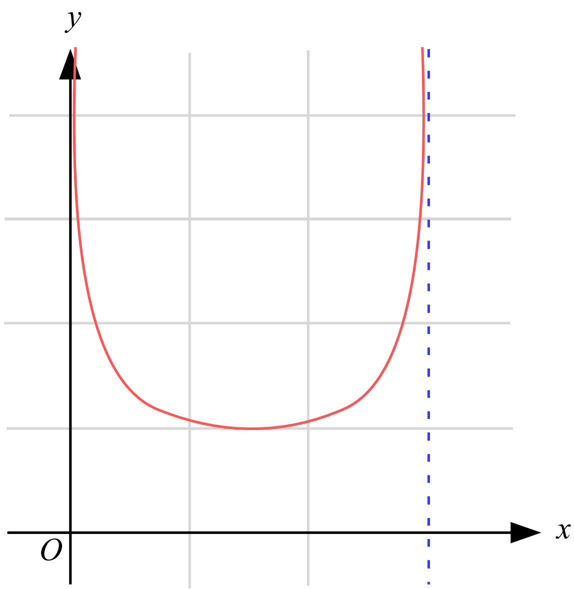
iii. Find the rule and domain of g^{-1}, the inverse of g. 2 marks

iv. The axes below show the graph of $y = g(x)$. Sketch the graph of $y = g^{-1}(x)$ on the same set of axes. Include exact coordinates of any endpoints, equations of any asymptotes and coordinates of any points of intersection correct to two decimal places. 2 marks



e. For what values of n does the tangent to the graph of $y = f\left(x\right)$ at the point $\left(n, f\left(n\right)\right)$ have x and y-intercepts which equal the same value? Give your answer correct to two decimal places.

3 marks

f. Let $k:(-5,1) \to \mathbb{R}, k(x) = \dfrac{3}{\sqrt{(x+5)(1-x)}}$.

The graph of k can be obtained by applying the transformation T to the graph of f where

$$T:\mathbb{R}^2 \to \mathbb{R}^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ n \end{bmatrix}$$

Find the value of m and the value of n.

2 marks

Question 2 (11 marks)

Andrew's heart rate can be modelled using the family of functions:

$$H(t) = k\,t\,e^{\left(-\frac{t}{k}\right)} + c$$

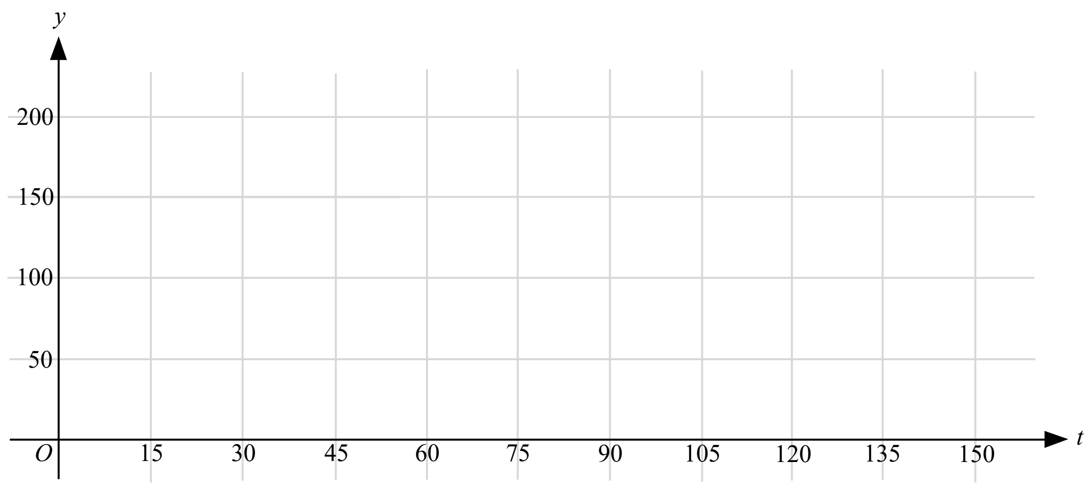
where $H(t)$ gives the heart rate in beats per minute (bpm) at time t minutes after a workout begins, $t \geq 0$, $k \in \mathbb{R}^+$ and $c \in \mathbb{R}^+$. Andrew's resting heart rate (at $t = 0$) is 65 bpm.

a. Show that $c = 65$. 1 mark

b. At what time, in terms of k, does the maximum heart rate for this session occur? 2 marks

c. Using the value of c from **part a** and $k = \sqrt{85e}$, find Andrew's maximum heart rate and the time, to the nearest minute correct to one decimal place, that this occurs. 2 marks

d. Given $H : [0, \infty) \to \mathbb{R}, H(t) = k\,t\,e^{\left(-\frac{t}{k}\right)} + 65$ and $k = \sqrt{85e}$, sketch the graph of $y = H(t)$ on the axes below. Include exact coordinates of any endpoints, stationary points and axis intercepts and the equations of any asymptotes. 2 marks



e. For the function in **part d**, how many minutes after reaching his maximum heart rate does Andrew's heart rate return to 70 bpm? State your answer correct to one decimal place. 2 marks

f. Andrew wants his maximum heart rate, of 150 bpm, to occur at $t = 20$ minutes. This can be modelled as the function $H^*(t)$, which is a transformation of $H(t)$, where his resting heart rate remains 65 beats per minute and $k = \sqrt{85e}$.

i. Describe the single dilation that transforms $H(t)$ into $H^*(t)$. 1 mark

ii. State the rule of $H^*(t)$. 1 mark

Question 3 (6 marks)

a. Given $h: \mathbb{R} \to \mathbb{R}, h(x) = \frac{x(x-25)^2}{500}$, and $g: [3, \infty) \to \mathbb{R}, g(x) = \sqrt{x^2 - 3x}$, state the range of $h \circ g$. 1 mark

b. Using the equation $y = h(x)$ from **part a**,

i. for what values of x is the gradient function of $h(x)$ strictly decreasing? 1 mark

ii. find the value of b for which $h(x) + b = 0$ has one solution. 1 mark

iii. find the value of b for which $h(x) + b = 0$ has three solutions. 1 mark

iv. find the real values of d for which only one of the solutions of $h(x+d) = \frac{3}{2}$ is positive. 2 marks

END OF SAC 1c

Mathematical Methods formula sheet

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1} x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		