| | Scotch Student ID # | | | |
|---|---|---|---|---|
| | | | | |
| Circle the relevant digits | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 1 |
| | 2 | 2 | 2 | 2 |
| | 3 | 3 | 3 | 3 |
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| Teacher's Name |
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# Scotch College

# MATHEMATICAL METHODS

## U4-SAC 1a – Application Task: Project

**Date of distribution: Monday 2nd August 2021**

**Due date: Wednesday 11th August 2021**

| Task Sections | Marks | Your Marks |
|---|---|---|
| Extended Response Questions | 60 | |
| **Total Marks** | **60** | |

**Remote Declaration**

*I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.*

*Signature:* _____

**General Instructions**

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

**Allowed Materials**

- A scientific calculator and a CAS calculator.
- Any notes or references.

**At the end of the task**

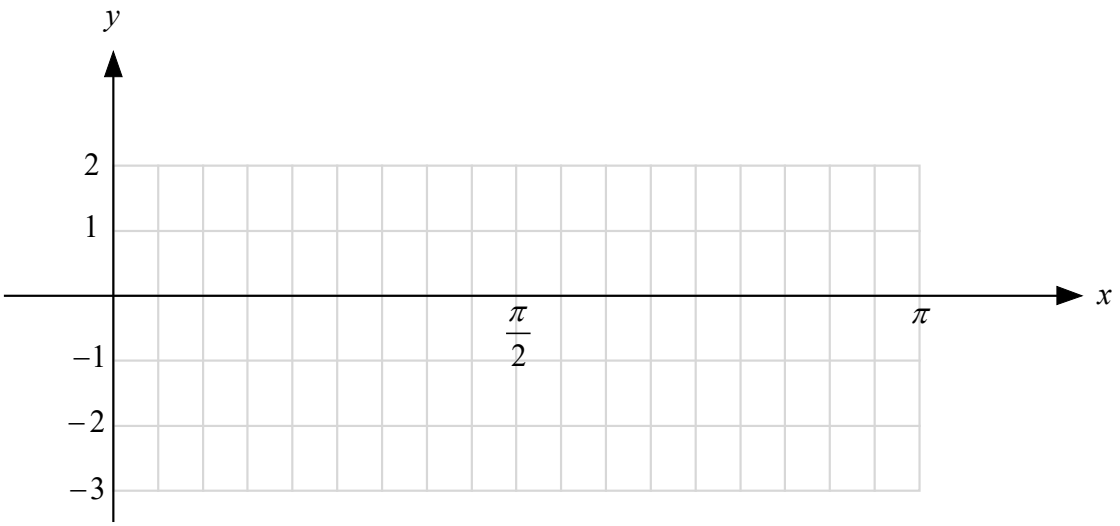
- Submit the task to your teacher by the due date and before the test SAC.

**Question 1** (11 marks)

**a.** Find the area of the regions bounded by the graph of $y = 2\sin(3x), 0 \le x \le \frac{2\pi}{3}$ and the $x$-axis. 2 marks

**b.** Let $f : [0, \pi] \to \mathbb{R}, f(x) = 2\sin(3x) - 1$.

**i.** Sketch the graph of $f$. Label axes intercepts, turning points and endpoints with their coordinates. 3 marks



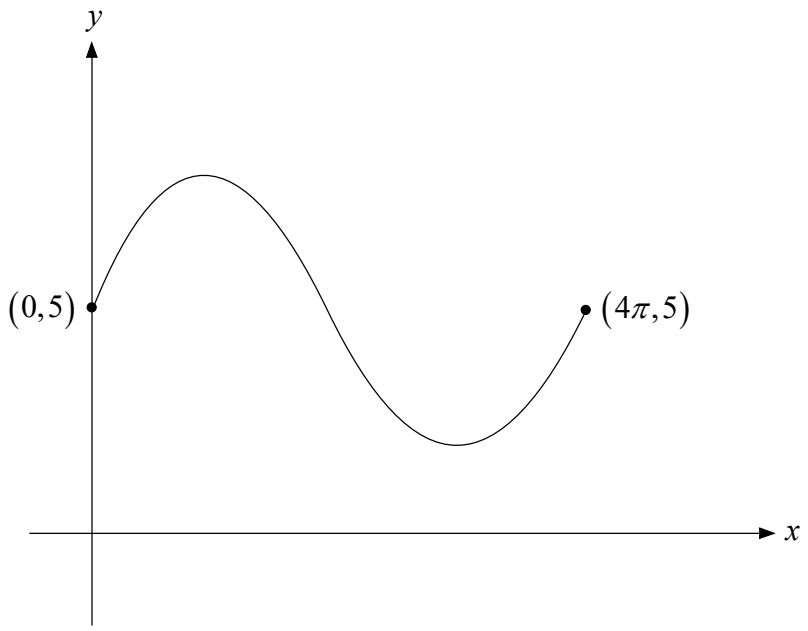

**ii.** Find the area of the regions bounded by the graph of $f$ and the $x$-axis. 3 marks

**c.** Find in terms of $n$ and $k$, the area of the region bounded by the $x$- and $y$-axes, the graph of $y = A\sin(nx) + k$, where $A > 0, n \in \mathbb{Z}^+$ and $k \geq A$, and

**i.** the line $x = \dfrac{2\pi}{n}$ 1 mark

**ii.** the line $x = 2\pi$ 1 mark

**d.** The graph drawn is a transformation of the graph with equation $y = \sin(x)$. Find the area of the region bounded by the graph, the $x$- and $y$-axes and the line $x = 4\pi$. 1 mark



**Question 2** (5 marks)

**a.** Find the area of the regions bounded by the graph of $y = 3\sin(x), \quad 0 \le x \le 2\pi$ and the $x$-axis. 2 marks

**b.** Find the value of $b$ $\left(b \in \mathbb{Z}^+\right)$ so that the area of the regions bounded by the graph of $y = 3\sin(bx), \quad 0 \le x \le \dfrac{2\pi}{b}$ and the $x$-axis is

**i.** 6 square units 1 mark

**ii.** 3 square units 1 mark

**c.** Find the values of $b$ $\left(b \in \mathbb{Z}^+\right)$ such that the area of the regions bounded by the graph of $y = 3\sin(bx), \quad 0 \le x \le 2\pi$ and the $x$-axis is 12 square units. 1 mark

**Question 3** (13 marks)

**a.** Find the area of the region bounded by the graphs of

$$y = \sin(x), \quad 0 \le x \le 2\pi$$
$$\text{and} \quad y = \cos(x), \quad 0 \le x \le 2\pi$$

3 marks

**b.** Find in terms of $b$, the area of the region bounded by the graphs of

$$y = \sin(bx), \quad 0 \le x \le \frac{2\pi}{b}$$
$$\text{and} \quad y = \cos(bx), \quad 0 \le x \le \frac{2\pi}{b}, \text{ where } b \in \mathbb{Z}^+$$

1 mark

**c.** Find in terms of $b$, the area of the regions bounded by the graphs of

$$y = \sin(bx), \quad 0 \le x \le 2\pi$$
$$\text{and} \quad y = \cos(bx), \quad 0 \le x \le 2\pi, \text{ where } b \in \mathbb{Z}^+$$

2 marks

**d.** Let $g:\left[0,\frac{2\pi}{b}\right]\to\mathbb{R}, g(x)=a\sin(bx)+a$ where $0<a\leq 2$ and $b\in\mathbb{Z}^+$,

**i.** find in terms of $a$, $b$ and $h$, the area of the region bounded by the graphs of

$$y=g(x)$$

and $y=g(x-h)$, where $0<h<\frac{\pi}{b}$ 4 marks

**ii.** find three sets of values for $a$, $b$ and $h$ when the area of the region found in **part d i** is $\sqrt{3}$ square units. 3 marks

**Question 4** (16 marks)

**a.** **i.** Find the value of $k$ so that the line $y = kx$ is tangent to the graph of $y = \sqrt{2x-3}$. 3 marks

**ii.** Using the value of $k$ found in **part a i**, find the area of the region enclosed by the tangent line $y = kx$, the graph of $y = \sqrt{2x-3}$ and the $x$-axis. 3 marks

**b.** **i.** Find the equation of the normal to the graph of $y = \sqrt{2x-3}$ at the point where $x = 3$. 1 mark

**ii.** Hence find the area of the region bounded by the normal, the curve and the $x$-axis. 3 marks

**c.** Let $f:\left[\frac{1}{a},\infty\right)\to\mathbb{R}, f(x)=\sqrt{ax-1}, \quad a>0.$

**i.** Find the value of $a$ so that the line $y=x$ is a tangent to the graph of $f$. 3 marks

**ii.** Using the value of $a$ found in **part c i**, find the area of the region bounded by the graphs of $f$, $f^{-1}$ and the $x$- and $y$-axes. 3 marks

**Question 5** (15 marks)

**a.** Let $f:\mathbb{R}\to\mathbb{R}, f(x)=e^{2x}+1,$

**i.** find the area of the region bounded by the graph of $f$, the $x$- and $y$-axes and the line $x=\frac{1}{2}\log_e(2)$ 2 marks

**ii.** find the rule for $f^{-1}(x),$ the inverse function of $f$ 2 marks

**iii.** find the area of the region enclosed by the graphs of $f$ and $f^{-1},$ the lines $x=3$ and $y=3,$ and the $x$- and $y$-axes. 2 marks

**b.** Find the average value of $f(x) = e^{2x} + 1$ for the interval $[0, 2]$. 2 marks

**c.** **i.** Find the area of the region bounded by the graphs of $y = e^{2x}$ and $y = 3e^{x} - 2$. 4 marks

**ii.** Let $g : \mathbb{R} \to \mathbb{R}, g(x) = e^{2x}$ and $h : \mathbb{R} \to \mathbb{R}, h(x) = ae^{x} - b$ where $a, b \in \mathbb{Z}^{+}$
find the relationship for *b* in terms of *a* so that the graphs of *g* and *h* have only one intersection point. 3 marks

**END OF SAC 2a**