



Scotch Student ID #				
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	7	7	7	7
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Teacher's Name

Scotch College
MATHEMATICAL METHODS

U4-SAC 1b – Application Task: Test REMOTE

Tuesday 24th August 2021

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	30	

Remote Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: _____

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are not allowed.
- Notes and/or references are not allowed.

At the end of the task

- Ensure you cease writing upon request.

Electronic Devices

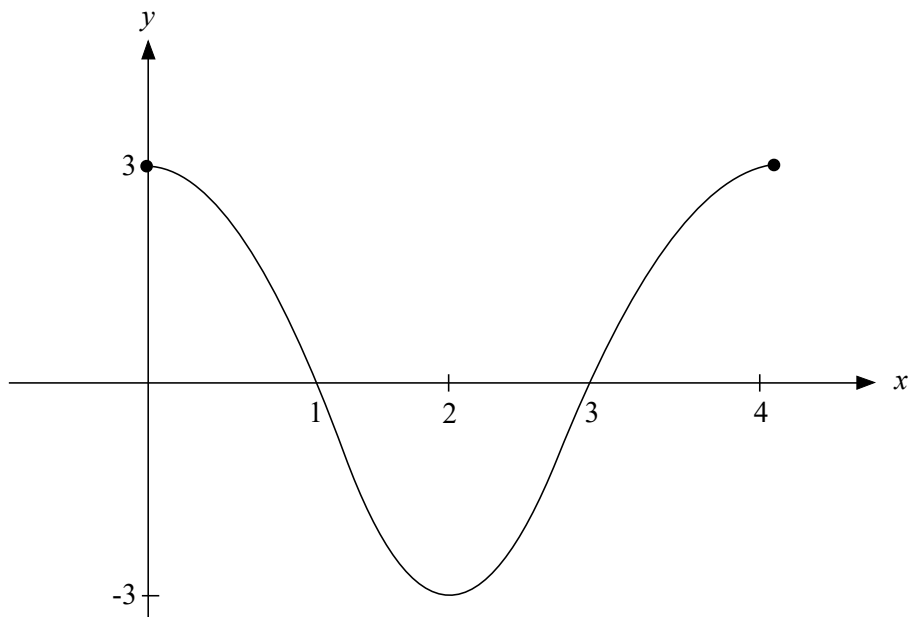
Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is **TURNT OFF** and is placed on the front teacher desk.

Question 1 (7 marks)

a. The equation of the graph below is of the form $y = A \cos(bx)$ for $0 \leq x \leq 4$.

Find the values of A and b .

2 marks



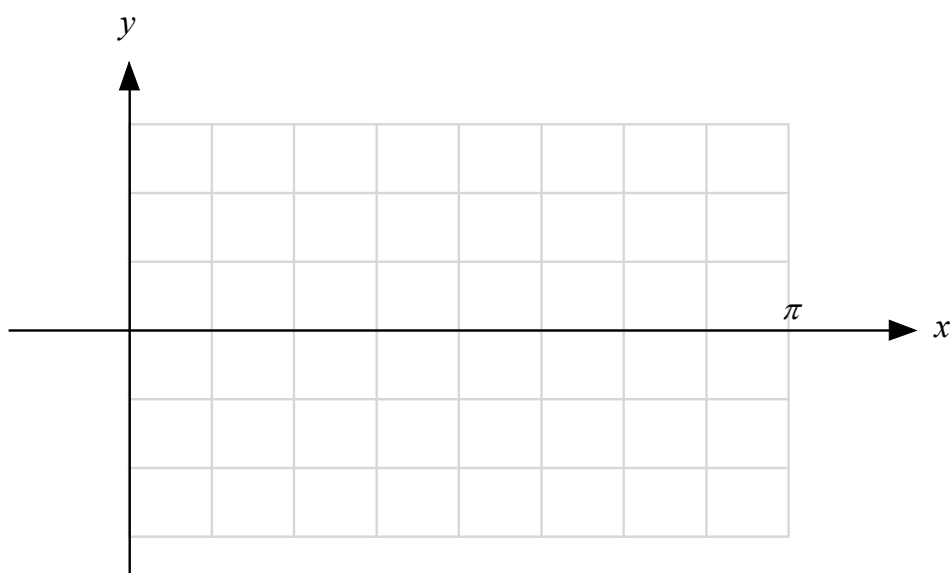
b. Let $f : [0, \pi] \rightarrow \mathbb{R}$, $f(x) = 1 - \sqrt{2} \cos(2x)$.

i. Solve $f(x) = 0$

2 marks

ii. Sketch the graph of f . Label axis intercepts, turning points and endpoints with their coordinates.

3 marks



Question 2 (5 marks)

- a. i.** Find the area of the region bounded by the graph of $y = 4 \sin(2x)$, $0 \leq x \leq \pi$ and the x -axis.

3 marks

- ii.** Hence or otherwise, find the area of the region bounded by the graph of $y = 4 \sin(x)$, $0 \leq x \leq 2\pi$ and the x -axis.

1 mark

- b.** Find the value of b ($b > 0$) such that the area of the region bounded by the graph of $y = 2 \sin(bx)$, $0 \leq x \leq \frac{2\pi}{b}$ and the x -axis is 4 square units.

1 mark

Question 3 (11 marks)

a. Find the area of the region bounded by the graphs of $y = \sqrt{3x-2}$ and $y = x$.

4 marks

b. Find the value of h such that the line $y = x$ is a tangent to the graph of $y = \sqrt{3x-h}$.

3 marks

- c. Find the area of the region bounded by the graphs of $y = x$ and $y = \sqrt{4x - 4}$, and the x -axis. 4 marks

Question 4 (7 marks)

Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = e^x - 2$

- a. Find the area of the regions bounded by the graph of f , the x -axis and the lines $x = 0$ and $x = \log_e(4)$.

3 marks

- b. Find, in terms of a , the area of the region bounded by the graphs of $y = e^x$ and $y = a + 1 - ae^{-x}$, where $a \in \mathbb{Z}^+$.

4 marks

END OF SAC 2b

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$		
$\frac{d}{dx}((ax+b)^n) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$		
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$		
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$		
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$		
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$		
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{p} = \frac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\text{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$