Circle the relevant digits	Scotch Student ID #			
	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

U4-SAC 2a – Application Task: Project

Date of distribution: Monday 30th August 2021

Due date: Monday 6th September 2021

Task Sections	Marks	Your Marks
Extended Response Questions	60	
Total Marks	**60**	

Remote Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

*Signature:*____

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- A scientific calculator and a CAS calculator.
- Any notes or references.

At the end of the task

- Submit the task to your teacher by the due date and before the test SAC.

Question 1 (12 marks)

The following table is the probability distribution of how many minutes an athlete must wait for a Covid-19 test each day at the Olympic Village. Let X be the random variable of the number of minutes the athlete must wait.

x	0	1	2	3	4	5
$\Pr(X = x)$	0.2	0.04	a	0.25	0.35	0.1

a. **i.** Show that the value of a is 0.06. 1 mark

ii. Evaluate the mean, μ, of the random variable X. 1 mark

iii. Evaluate the standard deviation, σ, of the random variable X. Give your answer correct to four decimal places. 1 mark

iv. Find $\Pr(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$. 2 marks

The queue for Covid-19 testing is managed by a security team. Four security teams supervise the queues in the proportions indicated in the table below, where $p > 0$.

Security team	1	2	3	4
Proportion of time on duty	$4q$	$p - q$	p^2	$p^2 + p$

b. **i.** Show that $2p(p+1) = 1 - 3q$. 1 mark

ii. If $q = \frac{1}{8}$, find the value of p and hence determine the probability that security team 4 was on duty. 2 marks

When security team 2 was on duty, the probability that someone tested positive was 0.04.

c. **i.** What is the probability, correct to three decimal places, that someone tested positive on at least one of the three most recent duties of team 2? 2 marks

ii. Find the least number of duties that security team 2 must complete, if the probability of someone testing positive on at least one duty is more than 0.3. 2 marks

Question 2 (14 marks)

At the Australian supporters' shop in Tokyo, data shows that the probability that a customer who enters the store purchases one or more souvenirs is 0.65.

a. On the first day of the Olympics, 384 customers enter the Australian supporters' shop.

Let the random variable X represent the number of customers who purchase one or more souvenirs.

i. Find the probability that the first four customers in the store **do not** purchase any souvenirs. Give your answer correct to three decimal places. 1 mark

ii. Find the mean number of customers on the first day who purchase at least one souvenir. 1 mark

iii. Find the probability, correct to six decimal places, that at least three quarters of the customers on the first day purchase at least one souvenir. 1 mark

b. On the second day, n customers enter the store. The manager calculates that the probability of at least thirty customers purchasing at least one souvenir is at least 98%.

Find the least such n. 2 marks

During the swimming finals, customers are particulary interested in purchasing Australian flags. The number of boxes of flags opened **each day** of the swimming finals, F, is a random variable with a distribution given by

F	0	1	2	3
$\Pr(F = f)$	0.1	0.35	0.25	0.3

c. Find the probability that a total of four boxes of flags had to be opened during the first two days of the swimming finals. Give your answer correct to four decimal places. 2 marks

The store has four cash registers for customers to complete their transactions. Customers randomly select a cash register but risk the EFTPOS machine needing to reset. The machine needing to reset on a given sale is independent of previous sales and resets. The store manager manager notes the following data from the first week of sales:

	Number of Sales	Number of times reset	Prob reset
Register 1	220	30	
Register 2	120	10	
Register 3	310	40	
Register 4	180	30	
Total			

d. Complete the table above by calculating the probability of the machine resetting and the totals. 2 marks

e. If five customers use register 4, find the probability that the register resets at least once. Give your answer correct to four decimal places. 2 marks

f. A customer visited the store on two occasions and made purchases on both days. Given that each register has an equal chance of being selected on each visit, find

i. the probability that they select the same register on both days 1 mark

ii. the probability that the register resets on exactly one of the two days. Give your answer correct to four decimal places. 2 marks

Question 3 (16 marks)

Four archers are warming up prior to their Olympic event.

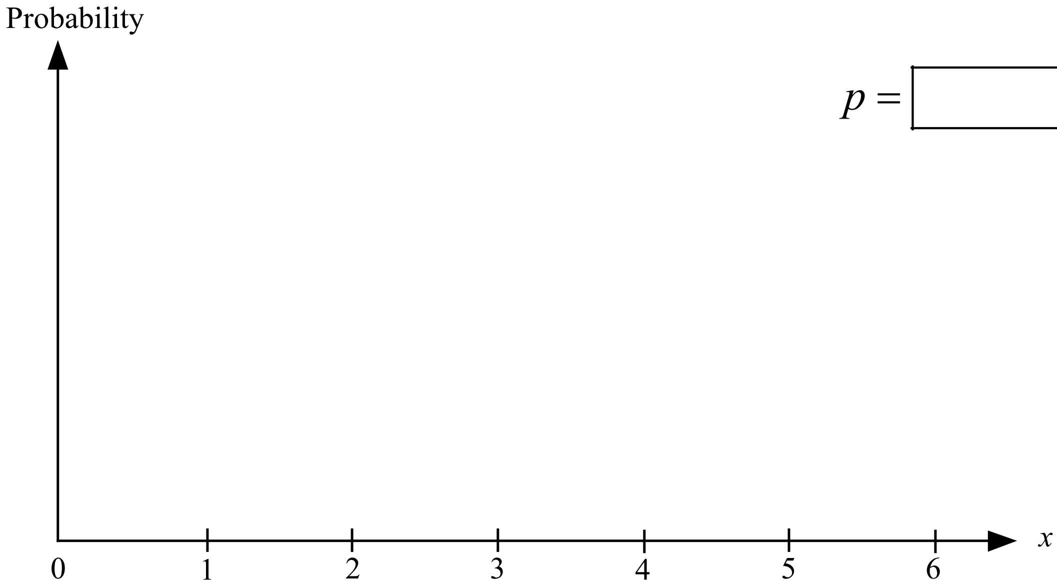
Archer *A* hits the target with probability 0.75 on each shot.

a. **i.** If he has six shots, find the probability of him hitting the target more than twice, correct to four decimal places. 1 mark

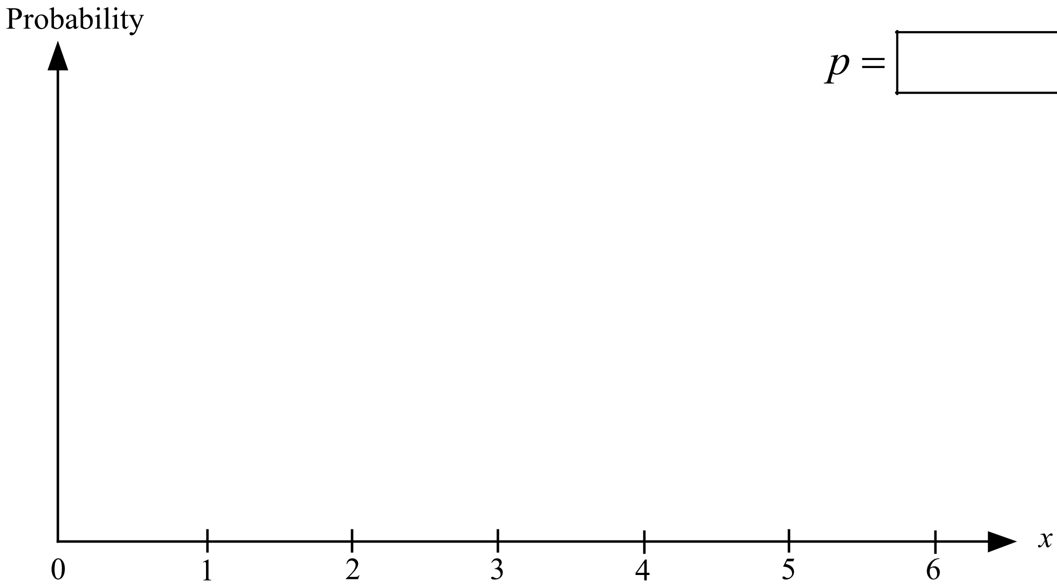
ii. Find the smallest number of shots needed to ensure a probability of more than 0.9 of hitting the target more than 5 times. 2 marks

Archer B has six shots, with a probability of p, of hitting a target.

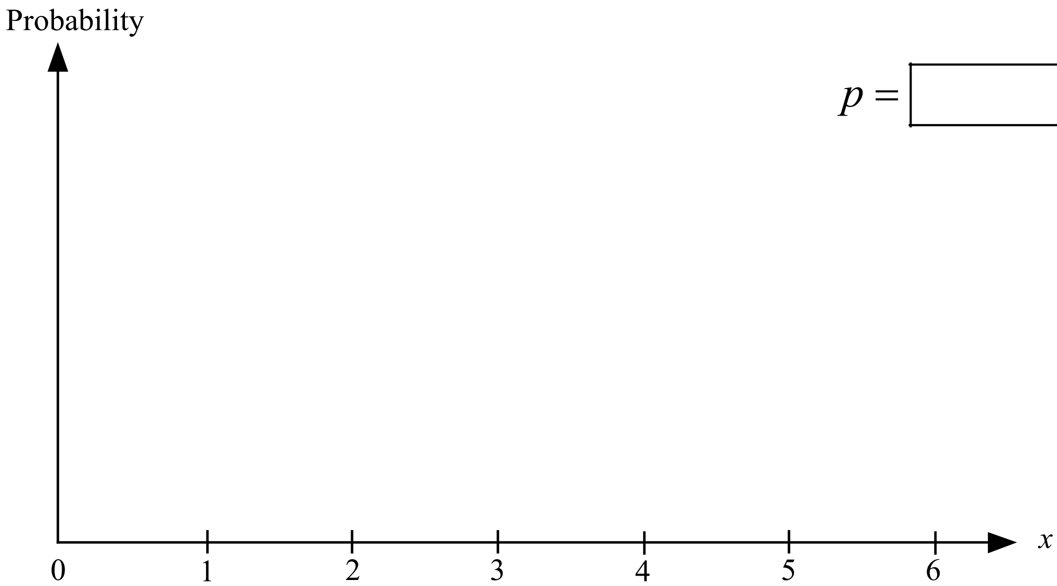
b. **i.** Sketch a graph of the probability distribution for Archer B for three different values of p on the three sets of axes below to demonstrate the different shapes the distribution can take. Clearly indicate your choice of p values.



$p =$ ☐ 1 mark



$p =$ ☐ 1 mark



$p =$ ☐ 1 mark

ii. Briefly comment on the effect of p on the shape of the distribution. 1 mark

iii. Archer B has six shots. If the probability of him hitting the target exactly once is 0.02048, find the value(s) of p, correct to five decimal places. 2 marks

Archer C has two bows he can choose from but he must use the same bow for all six shots.

Bow 1: When using Bow 1, he has a probability of 0.5 of hitting the target with each shot.

Bow 2: When using Bow 2, he has a probability of 0.85 of hitting the target on each of the first two shots and a probability of 0.3 for each of the next four shots.

c. **i.** Find the probability of hitting the target at least twice if he chooses Bow 1, correct to four decimal places. 1 mark

ii. Find the probability of hitting the target at least twice if he chooses Bow 2, correct to four decimal places. 2 marks

Archer D is very observant and notices that some arrows seem defective. Through his analysis he concludes that 5% of arrows are defective.

As part of quality control, he selects a random sample of 10 arrows each day throughout the Olympics for analysis, noting the number of defective arrows, each day.

d. i. On any given day, state the probability that he finds at least two defective arrows. Give your answer correct to four decimal places. 1 mark

ii. Find the probability that, in the first seven days, he finds two defective arrows on at least one day. Give your answer correct to four decimal places. 2 marks

iii. On one day, Archer D noted that there were four defective arrows. Should Archer D make an official complaint to the Olympic officials? Explain your reasoning. 1 mark

Question 4 (18 marks)

The probability that it rains at time T days since the Tokyo Olympic Games opened is given by

$$r(t)=\begin{cases}\lambda e^{-\lambda t}, & t\geq 0\\ 0 & , \text{ otherwise}\end{cases}$$

where $\lambda > 0$.

Note: The Tokyo Olympic Games officially opened at 9pm on 23rd July, 2021 AEST and ended on 8th August, 2021 and thus went for 16 days (that is, there are 16 24-hour periods).

a. Show that $r(t)$ is a probability density function. 2 marks

b. Find the probability that it rains during the Olympics in terms of λ. 2 marks

c. Show that an increase in λ increases the probability of rain in the first 16 days. 1 mark

d. **i.** Differentiate $\lambda t e^{-\lambda t}$ with respect to t. 1 mark

ii. Hence, find the expected value of T in terms of λ. 2 marks

e. If the probability of rain in the first $\dfrac{1}{p}$ days is p, find λ in terms of p. 2 marks

f. If $\lambda = \frac{1}{10}$, on which date is there a 50% chance that it would have already rained during the Olympics. 3 marks

g. The Olympic organisers have access to a marquee for 5 days and wish to use it when rain is most likely. That is, over the interval $[k, k+5]$. Find k such that $\Pr(k < T < k+5)$ is maximised for $\lambda = \frac{1}{10}$. 2 marks

h. Sketch a graph of r for $\lambda = \frac{1}{10}$ to show the probability of rain over the 16 days. Clearly indicate key features of your graph. 3 marks

END OF SAC 2a

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\ \cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\ \sin(ax)$		$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\text{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET