Circle the relevant digits	Scotch Student ID #			
	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

U4-SAC 2b – Application Task: Test REMOTE

Monday 6th September 2021

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	**30**	

Remote Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: ______________________________________

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are allowed.
- Notes and/or references are allowed.

At the end of the task

- Ensure you cease writing upon request.

Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (6 marks)

Let X represent the discrete random variable for the number of days that a swimmer competes in the Olympics over five days.

x	1	2	3	4	5
$\Pr(X = x)$	0.02	0.04	$8a$	$12a$	0.34

a. Show that $a = 0.03$ 1 mark

b. Find the probability that a swimmer competes on 3 or fewer days. 1 mark

c. Given that a swimmer competes on fewer than 5 days, find the probability that they swim on at least 2 days. Give your answer correct to four decimal places. 2 marks

d. Find the expected number of days that a swimmer will compete. 1 mark

e. Find the standard deviation for this distribution. Provide your answer correct to three decimal places. 1 mark

Question 2 (7 marks)

In the Olympic Village, each of Australia's 472 athletes choose where to eat their meals.

a. Research shows the probability an athlete chooses to eat breakfast in the Casual Dining Hall is 0.55, independent of where other athletes eat their breakfast.

i. Find the probability that the first three Australians who wake up choose the Casual Dining Hall. Give your answer correct to four decimal places. 1 mark

ii. Find the probability that five of the first seven Australians who wake up choose the Casual Dining Hall. Give your answer correct to four decimal places. 2 marks

iii. Find the expected number of Australian athletes who eat breakfast in the Casual Dining Hall each morning. 1 mark

b. All Australian athletes eat three meals per day. Traditional Japanese Food is offered at each meal and athletes choose this with the following probabilities:

Meal	Breakfast	Lunch	Dinner
Probability	0.1	0.6	0.8

i. Find the probability that an athlete chooses exactly two Japanese meals in one day. 2 marks

ii. Find the probability that at least half of the Australian athletes choose exactly two Japanese meals in one day. Give your answer correct to four decimal places. 1 mark

Question 3 (6 marks)

When warming up, the Australian Hockey team practice their penalty shots.

Player A has a probability of 0.7 of successfully shooting a goal.

a. Find the smallest number of shots needed to ensure the probability of shooting more than 4 goals is more than 0.9. 2 marks

b. A bag of hockey balls contains 25 white balls and 15 yellow balls. If Player A selects three balls at random without replacement, find the probability that he selects at least one ball of each colour. Give your answer correct to four decimal places. 2 marks

The coach is concerned about the quality of the balls. He concludes that 4% of balls are defective.

Each training session he analyses 20 balls.

c. Find the probability of finding at least two defective balls during a training session. Give your answer correct to four decimal places. 2 marks

Question 4 (11 marks)

The time, in seconds, between the winner crossing the finish line and the last athlete crossing the finishing line can be given by the continuous variable, T. The probability density function for T is given by

$$f(t)=\begin{cases}\dfrac{-2}{140625}t(t-15)(t^2+5t+20) & ,0\le t\le 15\\ 0 & ,\text{otherwise}\end{cases}$$

a. Find the probability that in a particular race, the last athlete finishes within 10 seconds of the winner. 1 mark

b. In total, six heats are held, all with the same probability density function. Find the probability that the last athlete finishes within 10 seconds of the winner in at least four of the heats. Give your answer correct to four decimal places. 2 marks

c. Determine the median time between the first and the last athlete crossing the finish line. Give your answer correct to two decimal places. 2 marks

d. State the expected time between the first and the last athlete crossing the line. 1 mark

e. Find the standard deviation for the time between the first and last athlete crossing the line. Give your answer correct to two decimal places. 2 marks

f. A special camera takes a burst of photos for 3 seconds. Find the 3 second interval of time after the first runner crosses the line that maximises the chance of capturing the last athlete as he crosses the finish line. Give your answer correct to four decimal places. 3 marks

END OF SAC 2b

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$		$\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x}\,dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$		$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$		$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET