Circle the relevant digits	Scotch Student ID #			
	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

U3-SAC 1a – Application Task: Project

Date of distribution: Wednesday 19th May 2021

Due date: Wednesday 2nd June 2021, prior to SAC 1b

Task Sections	Marks	Your Marks
Extended Response Questions	75	
Total Marks	**75**	

Remote Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: ____________________________________

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- A scientific calculator and a CAS calculator.
- Any notes or references.

At the end of the task

- Submit the task to your teacher by the due date.

Question 1 (6 marks)

A pair of equations are defined as follows:

$$(k+1)x - ky = 6$$
$$3x + 2ky + 4 = 0$$

where k is a real constant.

For what values of k do the pair of equations have:

a. a unique solution? 3 marks

b. no solutions? 2 marks

c. infinitely many solutions? 1 mark

Question 2 (5 marks)

The function f has rule $f(x) = x^{-\frac{1}{3}}$.

a. State the maximal domain for f. 1 mark

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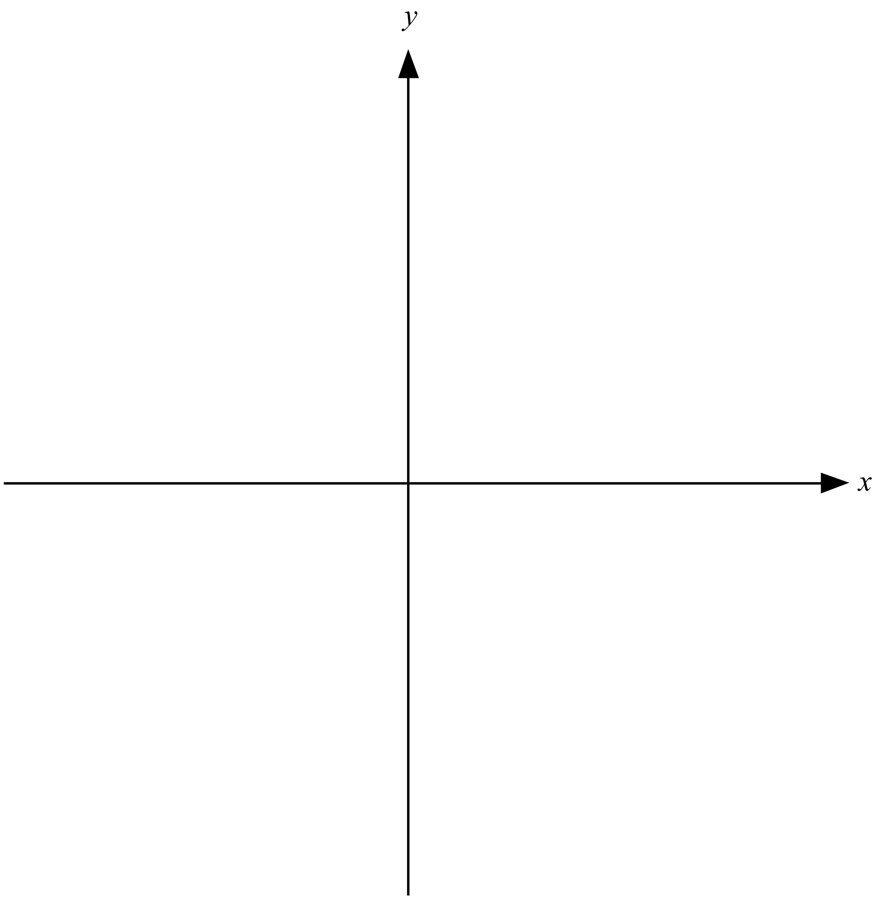
b. Sketch the graph of $y = f(x)$ on the axes below, labelling all intercepts, asymptotes and endpoints, if they exist. 3 marks



c. State the values of x for which the function f is strictly decreasing. 1 mark

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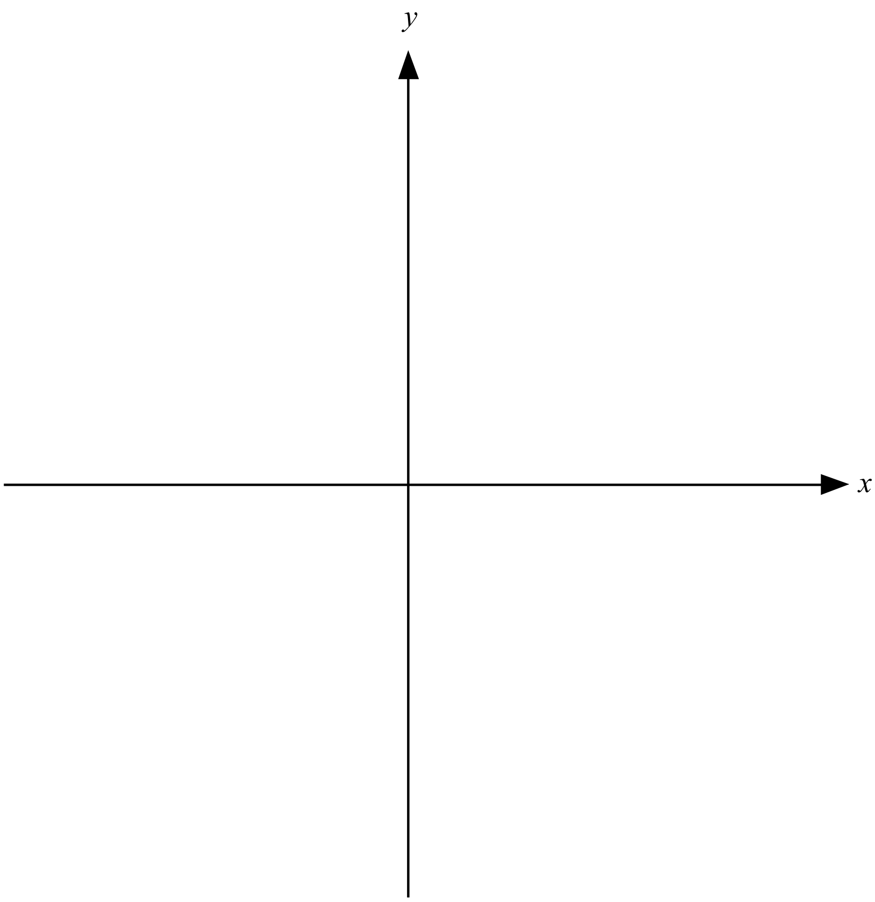
Question 3 (9 marks)

$f(x) = ax^3 + bx^2 + cx + d$, where a, b, c and d are real numbers.

a. The graph of $y = f(x)$ has stationary points at $(-3, 8)$ and $(5, 3)$. Find the rule $f(x)$. 2 marks

b. Sketch the graph of $y = f'(x)$ on the axes below, labelling all intercepts and the stationary point(s) with their coordinates. 2 marks



c. State the solutions to the equation $f'(x) = 0$. 1 mark

d. Consider another quadratic equation $x^2 + px + q = 0$, where p and q are real numbers and $p > 0, q > 0$.

i. Find the value(s) of p, in terms of q, for which this equation has exactly one solution. 1 mark

ii. Find the one solution to the equation in this case, in terms of q. 1 mark

e. The equation of a curve is $y = x^2 + px + q$, where p and q are real numbers.

i. For what values of p is the x-coordinate of the turning point of the curve positive? 1 mark

ii. Find the coordinates of the turning point of the curve in terms of p and q. 1 mark

Question 4 (13 marks)

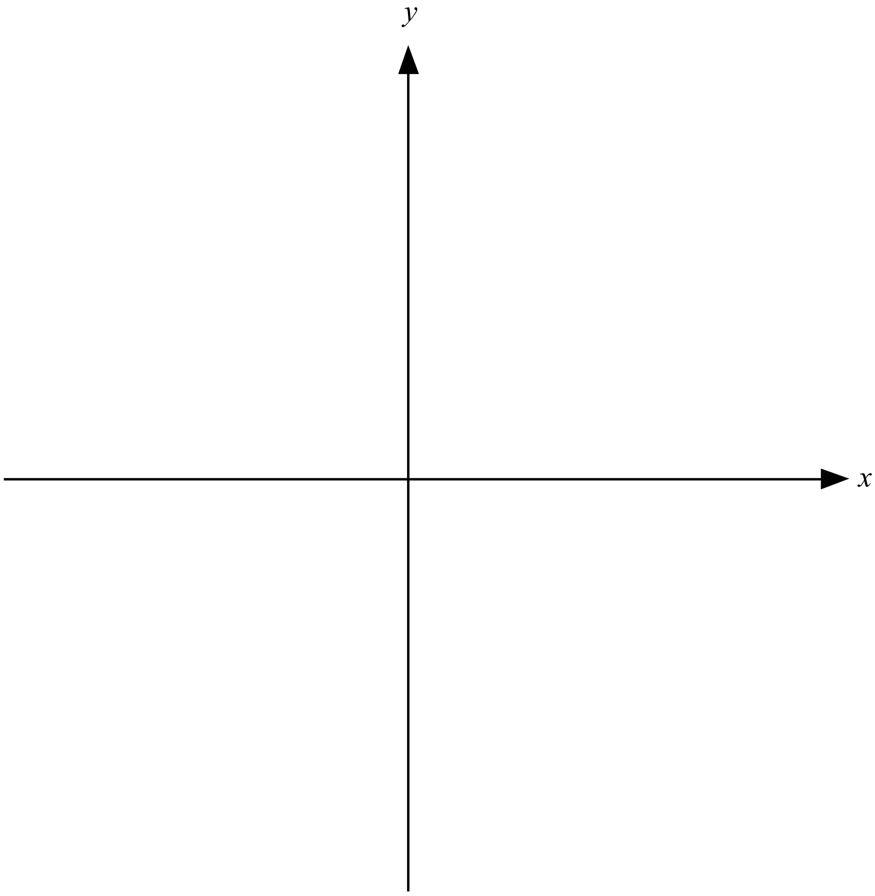
Two functions f and g are defined as:

$$f:\mathbb{R}\to\mathbb{R},\ f(x)=e^{3x}-2$$
$$g:\mathbb{R}\to\mathbb{R},\ g(x)=e^{x}$$

a. List a sequence of transformations which maps the graph of $y=g(x)$ to the graph of $y=f(x)$. 2 marks

b. Find a rule for the inverse, f^{-1}, of f and state its domain. 3 marks

c. Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes below, labelling all intercepts with exact coordinates, all asymptotes with their equations and the intersection point(s) between the curves to two decimal places. 4 marks



d. Fully define $h(x) = f\left(f^{-1}(x)\right)$. 2 marks

e. Show that $f\left(-2f^{-1}(3x)\right) = -\dfrac{18x^2+24x+7}{9x^2+12x+4}$. 2 marks

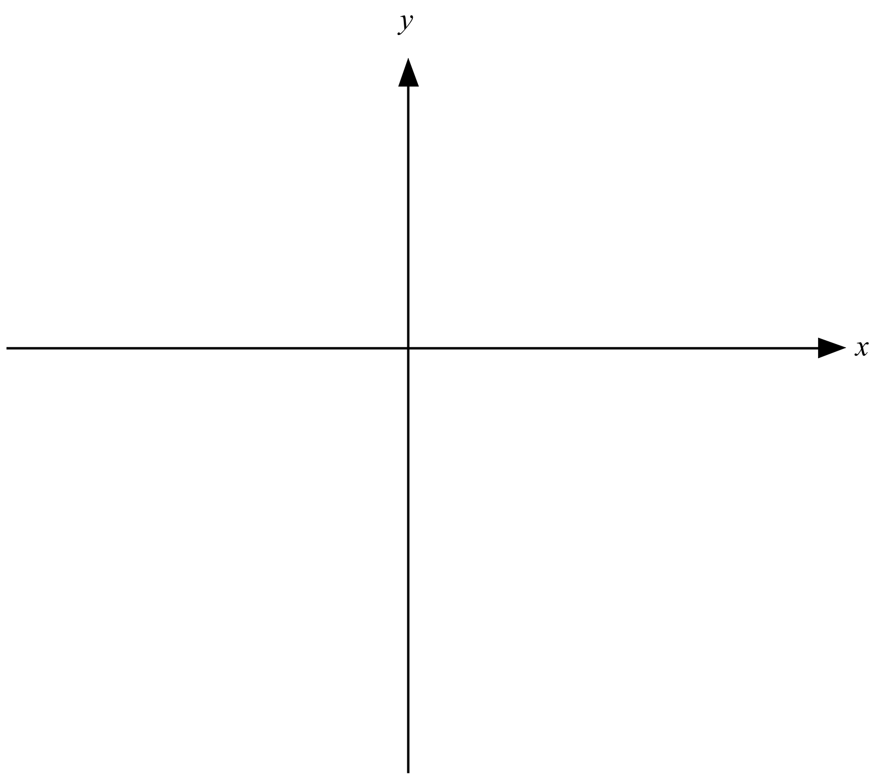
Question 5 (17 marks)

Consider the function f such that

$$f : D \to \mathbb{R}, f(x) = \frac{-1}{\sqrt{4-x^2}}$$

a. Find D, the maximal domain of f. 1 mark

b. Sketch the graph of $y = f(x)$ over its maximal domain. Label all key features. 3 marks



c. Hence, or otherwise, find the absolute maximum of f. 1 mark

d. Consider the function g such that

$$g : (-1, 4] \to \mathbb{R}, g(x) = -\sqrt{x+1}$$

On the above axes, sketch $y = g(x)$ including the exact coordinates of all endpoints and intercepts and the coordinates of the point(s) of intersection with $y = f(x)$ correct to two decimal places. 3 marks

e. Show that the function $f\left(g\left(x\right)\right)$ does not exist. 1 mark

f. Let $g^{*}: X \rightarrow \mathbb{R}, g^{*}(x) = -\sqrt{x+1}$ where X is the domain of g^{*}.

i. Find the maximal domain X such that $f\left(g^{*}\left(x\right)\right)$ exists. 1 mark

ii. Find the rule $f\left(g^{*}\left(x\right)\right)$. 1 mark

g. Let $p:(0,4)\rightarrow R, p(x)=\frac{2}{\sqrt{x(4-x)}}-1$. The graph of p can be obtained by applying the transformation T to the graph of f where

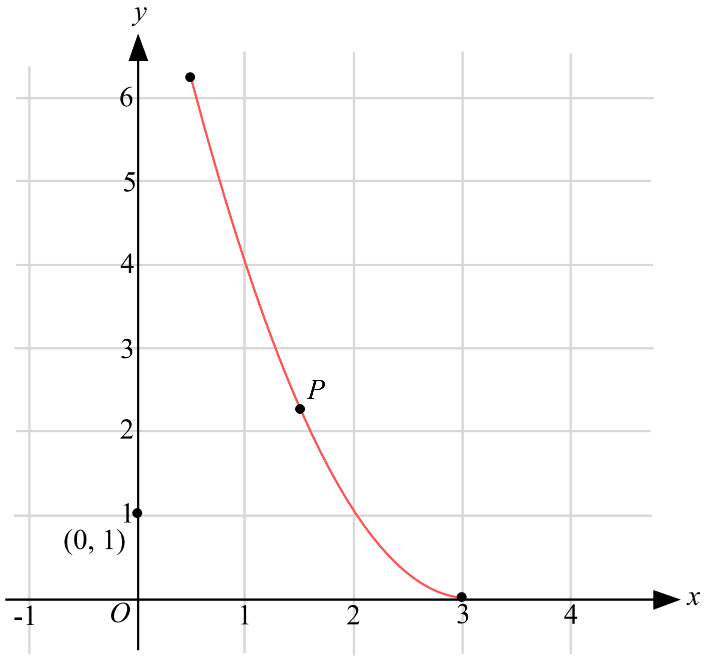
$$T: R^2 \rightarrow R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} 1 & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} q \\ n \end{bmatrix}$$

Find the values of m, n and q. 3 marks

h. Find the coordinates of the point P on the curve with equation $y=f(x)$ at which the tangent to $f(x)$ is normal to the line $3x+8y=15$, giving your answer correct to two decimal places. 3 marks

Question 6 (9 marks)

The curve $f(x)=(x-3)^2$ where $x\in\left[\frac{1}{2},3\right]$ is shown below. Point *P* on $f(x)$ has an *x*-value of $\frac{3}{2}$.



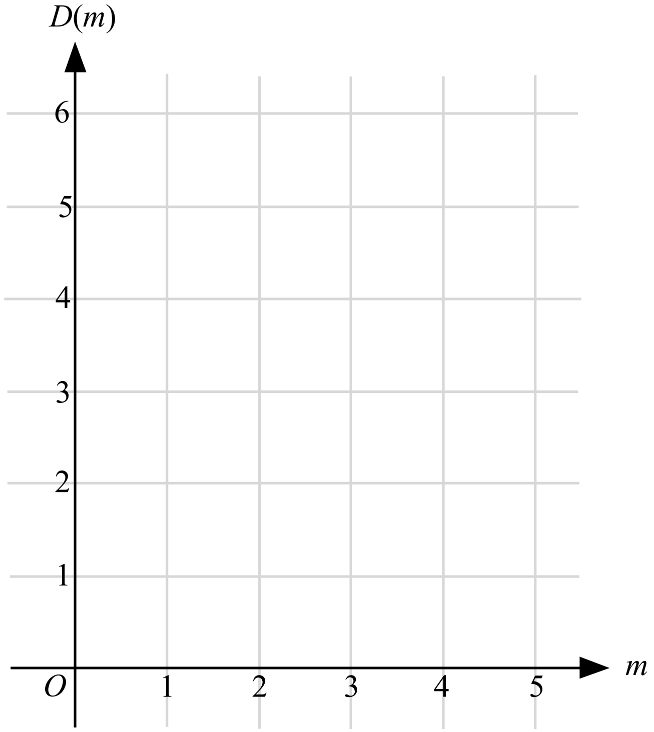
a. Calculate the distance from $(0,1)$ to Point *P*. 2 marks

b. Point *B* lies on $y=f(x)$ and has an *x*-coordinate of *m*. State the coordinates of point *B* in terms of *m*. 1 mark

c. Hence show that the distance $D(m)$ between $(0,1)$ and Point *B* is given by the function:

$$D(m)=\sqrt{m^2+(m-4)^2(m-2)^2}$$ 2 marks

d. Sketch $D(m)=\sqrt{m^2+(m-4)^2(m-2)^2}$ for $m \in \left[\frac{1}{2}, 3\right]$ on the axes below. Label the coordinates of all endpoints and stationary points, rounded to two decimal places. 2 marks



e. Find the value(s) of m for which $D(m)=4$, giving your answer(s) correct to two decimal places. 2 marks

Question 7 (16 marks)

A function $N(t)$ measures the intensity level of a treadmill workout *t* minutes after a workout begins. All workouts begin with the treadmill stationary, at intensity level zero, that is, $N(0) = 0$.

If the treadmill workout reaches an intensity level of 10 (that is, $N(t) = 10$) the treadmill enters the **override** mode. This means it will reduce the intensity level at a constant rate over a 5 minute period until $N(t) = 0$, and the treadmill stops.

$N(t) = \frac{1}{9000}(at - 10)^3(b - t) + 1$, where $t \geq 0$, $0 \leq N(t) \leq 10$, and a and b are real constants.

a. **i.** Show that $b = 9$. 2 marks

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ii. When $a = 0$, calculate the time taken, in minutes, to achieve the intensity level of 10. 2 marks

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iii. When $a = 2$, calculate the time taken after the workout has started for the intensity level, $N(t)$, to reduce to zero. State your answer in minutes correct to one decimal place. 2 marks

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iv. State, in terms of a, the possible value(s) of t for which $(t, N(t))$ is a stationary point of the function $N(t)$. 2 marks

v. For what value(s) of $a, a \in [0,6]$, does $N(t)$ have **no** stationary points? 1 mark

vi. For what value(s) of $a, a \in [0,6]$, does $N(t)$ have **one** stationary point? 2 marks

vii. What is the maximum number of stationary points $N(t)$ can have? 1 mark

b. **i.** What is the maximum intensity level achieved when $a = 6$ correct to one decimal place. 1 mark

ii. When $a = 6$, at what time is this maximum intensity level achieved? Give your answer in minutes, correct to one decimal place. 1 mark

c. a is restricted to the possible set of values below:

$$a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}.$$

What is the least value of $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, which will cause the treadmill to enter the **override** mode, and how many minutes in total will this workout last?

State your answer in minutes, correct to one decimal place. 2 marks

END OF SAC 1a

Mathematical Methods formula sheet

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x}\,dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		