Solutions

Scotch Student ID #				
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

Unit 3-SAC 1b – Application Task: Test

Wednesday 2nd June 2021

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	**30**	

Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: ____________________________________

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are not allowed
- Notes and/or references are allowed.

At the end of the task

- Ensure you cease writing upon request.

Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (9 marks)

A graph f has the rule $f(x) = (3x-5)^3$.

a. Find $f'(x)$. 2 marks

$f'(x) = 9(3x-5)^2$

b. Find the equation of the tangent to the graph of f at point $P(2,1)$. 2 marks

$y = mx + c$

$f'(2) = 9$

$y = 9x + c$

$1 = 9 \times 2 + c$

$c = -17$ $\therefore y = 9x - 17$

c. The tangent to the graph of f at point Q is parallel to the tangent to the graph of f at P.

Find the coordinates of point Q. 2 marks

$9(3x-5)^2 = 9$

$(3x-5)^2 = 1$

$3x - 5 = \pm 1$

$3x - 5 = 1$ or $3x - 5 = -1$

$3x = 6$ $3x = 4$

$x = 2$ $x = 4/3$

$(2, 1)$ $y = (3 \times \frac{4}{3} - 5)^3 = -1$

$\therefore (4/3, -1)$

d. List the sequence of transformations which maps the graph $y = (3x-5)^3$ to the graph $y = 2(5-x)^3 - 4$.

3 marks

- Dilated by a factor of 3 from the y-axis
- Dilated by a factor of 2 from the x-axis
- then reflected on y-axis
- then translated 10-units in the positive direction of x-axis
- then translated 4-units in the negative direction of y-axis.

Question 2 (3 marks)

A normal to the curve $y = \sqrt{x}$ has the equation $y = -8x + a$, where a is a real constant. Find the value of a.

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{8}$

$\therefore \quad \frac{1}{\sqrt{x}} = \frac{1}{4}$

$x = 16$

$y = 4$

$4 = -8(16) + a$

$4 = -128 + a$

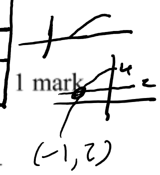
$a = 132$

Question 3 (10 marks)

Two curves f and g are defined as follows:

$$f:[2,\infty)\to\mathbb{R},\quad f(x)=\sqrt{x-2}$$
$$g:(-\infty,0]\to\mathbb{R},\quad g(x)=4-2x^2$$

	D	R
f	$[2,\infty)$	$[0,\infty)$
g	$(-\infty,0]$	$(-\infty,4]$



a. Is $g\circ f$ defined? Give reasons for your answer. 1 mark

$\text{Ran}(f)\not\subseteq \text{Dom}(g)$

$[0,\infty)\not\subseteq(-\infty,0]$

$\therefore$ $g\circ f$ does not exist

b. Find the domain of a suitably restricted function g^* of g such that $f\circ g^*(x)$ is defined on its maximal domain. 2 marks

$\text{Ran}(g)\not\subseteq\text{Dom}(f)$

$(-\infty,4]\not\subseteq[2,\infty)$

$[2,4]\subseteq[2,\infty)$

$\therefore$ Domain $[-1,0]$

$4-2x^2=2$

$2=2x^2$

$x^2=1$

$x=\pm1$

c. **i.** Find the rule for g^{-1}, the inverse of g. 2 marks

Let $y=g(x)$

$y=4-2x^2$

$x=4-2y^2$

$2y^2=4-x$

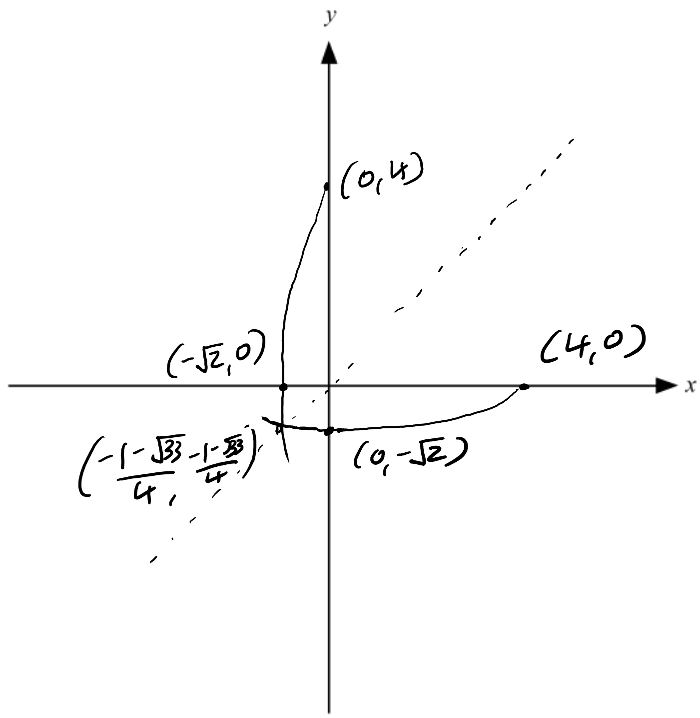
$y=\pm\sqrt{\frac{4-x}{2}}$

$\therefore g^{-1}(x)=-\sqrt{\frac{4-x}{2}}$

ii. State the domain of g^{-1}. 1 mark

$x \in (-\infty, 4]$

d. Sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$ on the same set of axes below, showing all intersections, endpoints and intercepts. You may use the lines below for working. 3 marks



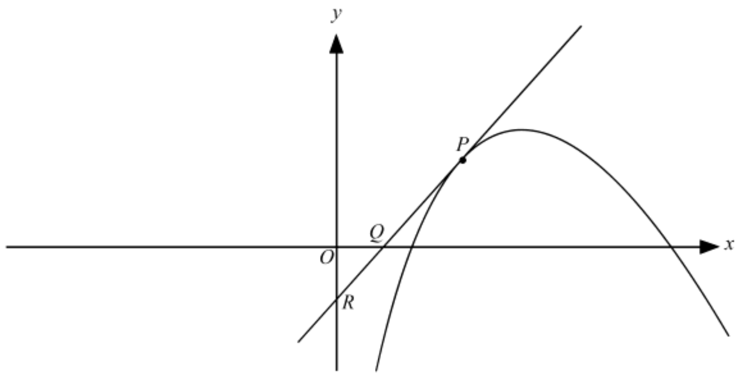
$x = 4 - 2x^2$

$2x^2 + x - 4 = 0$

$x = \frac{-1 \pm \sqrt{1+32}}{4} = \frac{-1 \pm \sqrt{33}}{4}$

Question 4 (4 marks)

The curve with rule $f(x) = 6\log_e(x) - 4x^{\frac{1}{2}}$ and the tangent to the curve at point P are shown below:



Point P has an x-coordinate of 4. The tangent to the graph of $y = f(x)$ at P crosses the x-axis at Q and the y-axis at R. Find a and b if the area of the triangle OQR is $(a + b\log_e 2)^2$.

$f'(x) = \frac{6}{x} - \frac{2}{\sqrt{x}}$

$f'(4) = \frac{3}{2} - 1 = 1/2$

$y = \frac{1}{2}x + c$

$6\log_e(4) - 8 = 2 + c$

$c = 6\log_e(4) - 10$

$\therefore y = \frac{1}{2}x + 6\log_e(4) - 10$

y-int $\Rightarrow x = 0$

$\therefore y = 6\log_e(4) - 10$

x-int $\Rightarrow y = 0$

$0 = \frac{x}{2} + 6\log_e(4) - 10$

$\therefore x = 20 - 12\log_e(4)$

$A = \frac{(20 - 12\log_e(4)) \times (10 - 6\log_e(4))}{2}$

$= (10 - 6\log_e(4))^2 = (10 - 12\log_e(2))^2$

$\therefore a = 10 \quad b = -12$

Question 5 (5 marks)

Let b, c, p and q be real numbers.

a. Consider the equation $0 = x^2 + bx + c$, where $c > 0$.

i. Find the value(s) of b for which the equation has two distinct real solutions, giving your answer(s) in terms of c. 2 marks

$\Delta > 0$

$\therefore\ b^2 - 4c > 0$

$(b - 2\sqrt{c})(b + 2\sqrt{c}) > 0$

$\therefore\ b \in (-\infty, -2\sqrt{c}) \cup (2\sqrt{c}, \infty)$

ii. Determine the range of values of b for which both distinct real solutions are positive. Justify your answer. 1 mark

$x^2 + bx + (\frac{b}{2})^2 - (\frac{b}{2})^2 + c$

$(x + \frac{b}{2})^2 - \frac{b^2}{4} + c$

$-\frac{b}{2} < 0$

$b < 0$

$-\frac{b^2}{4} + c < 0$

$(2\sqrt{c} - b)(2\sqrt{c} + b) < 0$

$b < -2\sqrt{c}$

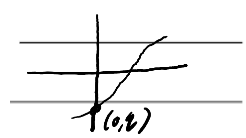
b. Consider the equation $x^3 + px + q = 0$, where $p > 0$ and $q < 0$. Find the number of solutions to this equation and state the sign(s), justifying your answer. 2 marks

Let $y = x^3 + px + q$

$\frac{dy}{dx} = 3x^2 + p = 0$

$3x^2 = -p$

$x^2 = \frac{-p}{3}$ since $p > 0$



There are no st. points and one positive solution

END OF SAC 1b