	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

Unit 3-SAC 1b – Application Task: Test

Wednesday 2nd June 2021

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	**30**	

Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: _____

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are not allowed
- Notes and/or references are not allowed.

At the end of the task

- Ensure you cease writing upon request.

Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (9 marks)

A graph f has the rule $f(x) = (3x-5)^3$.

a. Find $f'(x)$. 2 marks

b. Find the equation of the tangent to the graph of f at point $P(2,1)$. 2 marks

c. The tangent to the graph of f at point Q is parallel to the tangent to the graph of f at P.

Find the coordinates of point Q. 2 marks

d. List the sequence of transformations which maps the graph $y = (3x - 5)^3$ to the graph $y = 2(5 - x)^3 - 4$. 3 marks

Question 2 (3 marks)

A normal to the curve $y = \sqrt{x}$ has the equation $y = -8x + a,$ where a is a real constant. Find the value of a.

Question 3 (9 marks)

Two curves f and g are defined as follows:

$$f:[2,\infty)\to\mathbb{R},\quad f(x)=\sqrt{x-2}$$
$$g:(-\infty,0]\to\mathbb{R},\quad g(x)=4-2x^2$$

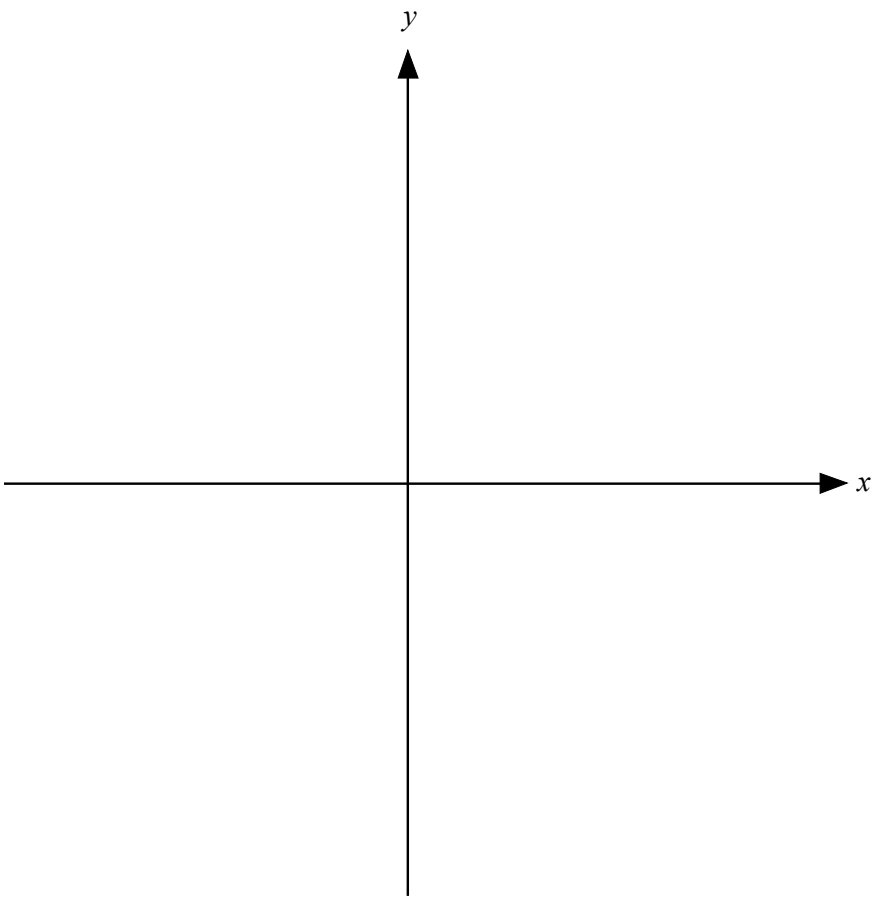
a. Is $g\circ f$ defined? Give reasons for your answer. 1 mark

b. Find the domain of a suitably restricted function g^* of g such that $f\circ g^*(x)$ is defined on its maximal domain. 2 marks

c. **i.** Find the rule for g^{-1}, the inverse of g. 2 marks

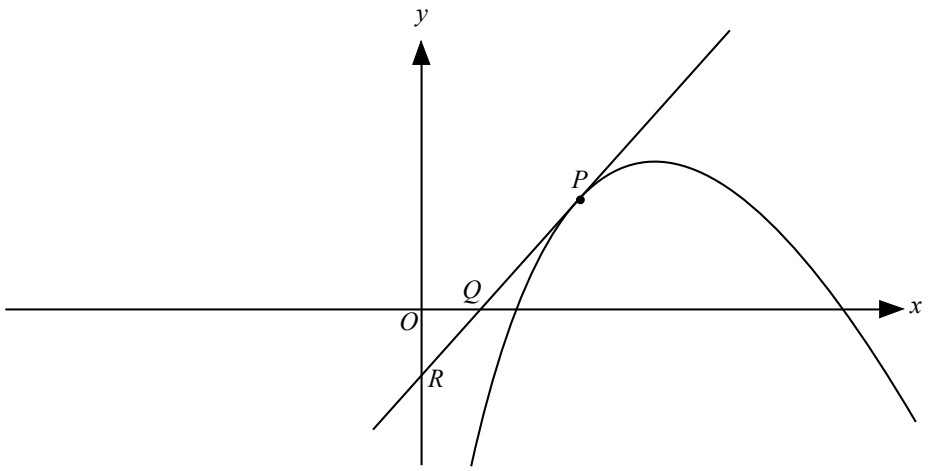
ii. State the domain of g^{-1}. 1 mark

d. Sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$ on the same set of axes below, showing all intersections, endpoints and intercepts. You may use the lines below for working. 3 marks



Question 4 (4 marks)

The curve with rule $f(x) = 6\log_e(x) - 4x^{\frac{1}{2}}$ and the tangent to the curve at point P are shown below:



Point P has an x-coordinate of 4. The tangent to the graph of $y = f(x)$ at P crosses the x-axis at Q and the y-axis at R. Find a and b if the area of the triangle OQR is $(a + b\log_e 2)^2$.

Question 5 (5 marks)

Let b, c, p and q be real numbers.

a. Consider the equation $0 = x^2 + bx + c$, where $c > 0$.

i. Find the range of values of b for which the equation has two distinct real solutions, giving your answer in terms of c. 2 marks

ii. Determine the range of values of b for which both distinct real solutions are positive, giving your answer in terms of c. Justify your answer. 1 mark

b. Consider the equation $x^3 + px + q = 0$, where $p > 0$ and $q < 0$. Find the number of solutions to this equation and state the sign(s), justifying your answer. 2 marks

END OF SAC 1b

Mathematical Methods formula sheet

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c, \ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		