	Scotch Student ID #			
Circle the relevant digits				
	0	0	0	0
	1	1	1	1
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

Unit 3-SAC 1c – Application Task: Test

Wednesday 2nd June 2021

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	**30**	

Declaration
I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.   *Signature:* ___________________________________

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are allowed
- Notes and/or references are not allowed.

At the end of the task

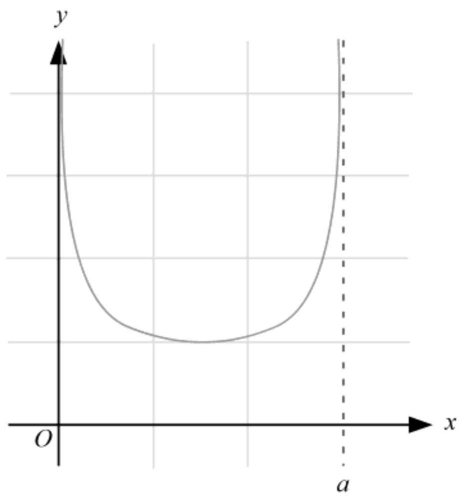
- Ensure you cease writing upon request.

Electronic Devices

Students are **not** allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (13 marks)

The graph of the function $f:(0,a)\to\mathbb{R}, f(x)=\dfrac{3}{2\sqrt{3x-x^2}}$, including asymptotes at $x=0$ and $x=a$ is shown below.



a. Show that $a=3$. 1 mark

$3x-x^2>0$

$x(3-x)>0$

$\therefore \quad 0<x<3$

$\therefore \quad a=3$

b. Write down the equations of asymptotes of the curve $y=f\left(\frac{x}{2}\right)+3$. 1 mark

$x=0, \; x=6$

c. Let $g:(0,c]\to\mathbb{R}, g(x)=f(x)$, where c is the largest value of x such that g has an inverse function.

i. State c. 1 mark

$c=3/2$

ii. Hence or otherwise, find the absolute minimum of f. 1 mark

Absolute minimum of f is 1

iii. Find the rule and domain of g^{-1}, the inverse of g. 2 marks

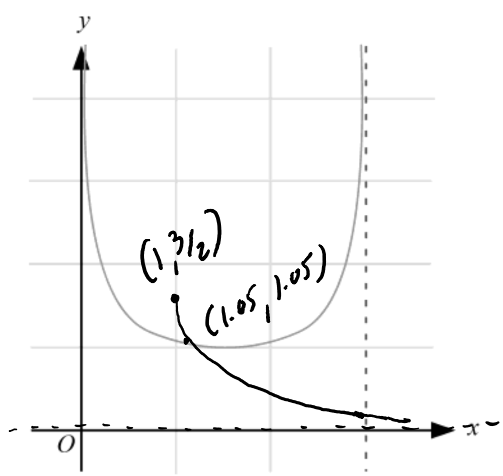
Let $y = g(x)$

$y = \dfrac{3}{2\sqrt{3x - x^2}}$ $\quad$ $y = \dfrac{3x - 3\sqrt{x^2 - 1}}{2x}$

$x = \dfrac{3}{2\sqrt{3y - y^2}}$ $\quad$ $g^{-1}(x) = \dfrac{3x - 3\sqrt{x^2 - 1}}{2x}$

$\text{Dom } g^{-1} = [1, \infty)$

iv. The axes below show the graph of $y = g(x)$. Sketch the graph of $y = g^{-1}(x)$ on the same set of axes. Include exact coordinates of any endpoints, equations of any asymptotes and coordinates of any points of intersection correct to two decimal places. 2 marks



Asy $y = 0$

e. For what values of n does the tangent to the graph of $y = f(x)$ at the point $(n, f(n))$ have x and y-intercepts which equal the same value? Give your answer correct to two decimal places. 3 marks

$$y = \frac{-3(2n-3)}{4n(n-3)\sqrt{3n-n^2}}x + \frac{3(4n-9)}{4(n-3)\cdot\sqrt{-n(n-3)}}$$

$$\text{y-int} \Rightarrow \frac{3(4n-9)}{4(n-3)\sqrt{-n(n-3)}}$$

$$\text{x-int} \Rightarrow \frac{n(4n-9)}{2n-3}$$

solve for y-int = x-int

$n = 0.521141557$ or $n = \frac{9}{4}$

as $0 < n < 3/2$ $\therefore n \approx 0.52$

method-2

$y = -x$

$f'(x) = -1$

$x = 0.521141557$

$\therefore n \simeq 0.52$

f. Let $k: (-5,1) \to \mathbb{R}, k(x) = \dfrac{3}{\sqrt{(x+5)(1-x)}}$.

The graph of k can be obtained by applying the transformation T to the graph of f where

$$T: \mathbb{R}^2 \to \mathbb{R}^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ n \end{bmatrix}$$

Find the value of m and the value of n. 2 marks

$x' = -2x + 1$ $\qquad x = \dfrac{1-x'}{2}$

$y' = my + n$ $\qquad y = \dfrac{y'-n}{m}$

$$y = \frac{3}{2\sqrt{3x - x^2}}$$

$$\frac{y'-n}{m} = \frac{3}{2\sqrt{3\left(\frac{1-x'}{2}\right) - \left(\frac{1-x'}{2}\right)^2}}$$

$$y' = \frac{3m}{\sqrt{(x'+5)(1-x')}} + n$$

$\therefore m = 1$

& $n = 0$

Question 2 (11 marks)

Andrew's heart rate can be modelled using the family of functions:

$$H(t) = k\,t\,e^{\left(-\frac{t}{k}\right)} + c$$

where $H(t)$ gives the heart rate in beats per minute (bpm) at time t minutes after a workout begins, $t \geq 0$, $k \in \mathbb{R}^+$ and $c \in \mathbb{R}^+$. Andrew's resting heart rate (at $t = 0$) is 65 bpm.

a. Show that $c = 65$. 1 mark

$H(0) = 65 \quad \therefore 65 = k(0)e^0 + c$

$\therefore c = 65$

b. At what time, in terms of *k*, does the maximum heart rate for this session occur? 2 marks

$H'(t) = ke^{-t/k} - te^{-t/k} = 0$

$\therefore t = k$

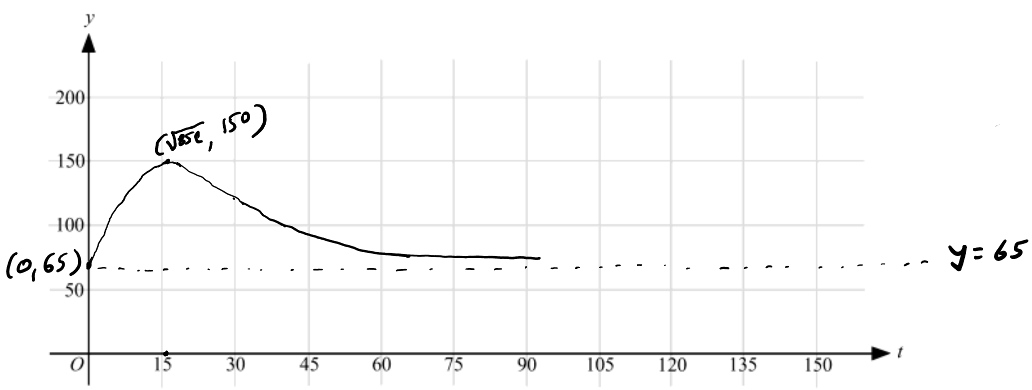
c. Using the value of *c* from **part a** and $k = \sqrt{85e}$, find Andrew's maximum heart rate and the time, to the nearest minute, that this occurs. 2 marks

$H(t) = \sqrt{85e}.t.e^{-t/\sqrt{85e}} + 65$

$H'(t) = 0 \quad \therefore t = 15$ mins

$H(\sqrt{85e}) = 150$

d. Given $H : [0, \infty) \to \mathbb{R}$, $H(t) = k\,t\,e^{\left(-\frac{t}{k}\right)} + 65$ and $k = \sqrt{85e}$, sketch the graph of $y = H(t)$ on the axes below. Include exact coordinates of any endpoints, stationary points and axis intercepts and the equations of any asymptotes. 2 marks



e. For the function in **part d**, how many minutes after reaching his maximum heart rate does Andrew's heart rate return to 70 bpm? State your answer correct to one decimal place. 2 marks

$H(t) = 70$ solve for t

$t = 0.3$ or $t = 84.3$

so after reaching max

$84.3 - \sqrt{85e} = 69.1$ mins

f. Andrew wants his maximum heart rate, of 150 bpm, to occur at $t = 20$ minutes. This can be modelled as the function $H^*(t)$, which is a transformation of $H(t)$, where his resting heart rate remains 65 beats per minute and $k = \sqrt{85e}$.

i. Describe the single dilation that transforms $H(t)$ into $H^*(t)$. 1 mark

Dilation by a factor $\frac{20}{\sqrt{85e}}$ from the y-axis.

ii. State the rule of $H^*(t)$. 1 mark

$H^*(t) = \frac{85e}{20} \cdot t \cdot e^{-t/20} + 65$

$= \frac{17}{4} \cdot t \cdot e^{-t/20} + 65$

SAC 1c - continued
TURN OVER

Question 3 (6 marks)

a. Given $h:\mathbb{R}\to\mathbb{R}, h(x)=\frac{x(x-25)^2}{500}$, and $g:[3,\infty)\to\mathbb{R}, g(x)=\sqrt{x^2-3x}$, state the range of $h\circ g$. 1 mark

$[0,\infty)$

b. Using the equation $y=h(x)$ from **part a**,

i. for what values of x is the gradient function of $h(x)$ strictly decreasing? 1 mark

$x \in \left(-\infty \ \frac{50}{3}\right]$

ii. find the value of b for which $h(x)+b=0$ has one solution. 1 mark

$b>0$ or $b<-\frac{125}{27}$

iii. find the value of b for which $h(x)+b=0$ has three solutions. 1 mark

$-\frac{125}{27}<b<0$

iv. find the real values of d for which only one of the solutions of $h(x+d)=\frac{3}{2}$ is positive. 2 marks

Solve for $h(x)=3/2$

$x=10-5\sqrt{3},\ 10+5\sqrt{3},\ 30$

$\therefore\ 10+5\sqrt{3}\leq d<30$

END OF SAC 1c