



Scotch Student ID #				
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Teacher's Name

Scotch College
MATHEMATICAL METHODS

U4-SAC 2a – Application Task: Project

Date of distribution: Thursday 1st September 2022

Due date: Thursday 8th September 2022

Task Sections	Marks	Your Marks
Extended Response Questions	60	
Total Marks	60	

Remote Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: _____

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- A scientific calculator and a CAS calculator.
- Any notes or references.

At the end of the task

- Submit the task to your teacher by the due date and before the test SAC.

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Question 1 (11 marks)

With final exams rapidly approaching, Year 12 Scotch College boys are planning how they will relax and spend their time once the exams are over. Many of the boys will holiday in a group of friends. The following table shows the probability distribution for the number of boys, X , in a randomly selected group.

x	2	3	4	5	6	7	8
$\Pr(X = x)$	0.05	a	0.35	0.26	0.07	0.1	0.14

- a.** Show that $a = 0.03$ 1 mark

- b.** Find the probability that a randomly chosen group will have more than four boys in it. 2 marks

- c.** What is the expected number of boys in a group? 2 marks

- d. Find $E(X^2)$ and hence determine the standard deviation of the number of boys in a group, correct to two decimal places.

2 marks

- e. Given that a randomly selected group contains at least three boys, what is the probability that the group contains no more than five boys?

2 marks

- f. Fifteen randomly chosen Year 12 students are surveyed about what size group they will be holidaying in. What is the probability that more than half of them will be holidaying in a group of exactly four students? Give your answer correct to four decimal places.

2 marks

Question 2 (10 marks)

One group of boys intends to relax after exams by inventing and playing games. One such game involves rolling a biased six-sided die twice and noting the sum of the results.

The die is biased so that the probability of rolling a one is $\frac{x}{4}$, the probability of rolling a two is $\frac{1}{4}$, the probabilities of rolling a three, four or five are each $\frac{1}{6}$ and the probability of rolling a 6 is $\frac{1}{4}(1-x)$.

- a. What is the probability of rolling a sum of two? Give your answer in terms of x . 1 mark

- b. Show that the probability of rolling a sum of seven is given by $\frac{10+9x-9x^2}{72}$. 2 marks

- c. Determine the value of x that maximises the probability of rolling a sum of seven and calculate this probability. 3 marks

Through empirical methods, the boys have determined that the value of x is actually $\frac{1}{4}$. Using this value of x , they have devised a game that costs \$3.00 to enter with a prize of \$10.00 if a sum of seven is obtained and \$1.00 if any other sum is obtained.

Note - The boys are too smart to gamble with real money, so they decide to use Monopoly money.

- d.** What is the expected gain/loss for a student playing the game, correct to the nearest cent? 3 marks

- e.** Is this game fair? Explain your answer. 1 mark

Question 3 (5 marks)

One boy is going away with six friends after the exams, making a group of seven. He starts to plan what music to download for them to listen to. At school he does a quick survey and determines that 45% of Year 12 students like hip-hop music.

a. If he downloads hip-hop music to take on holiday, what is the probability, correct to three decimal places, that

i. exactly three of his six friends will like it? 2 marks

ii. no more than two of his six friends will like it? 1 mark

b. What would be the smallest number of friends he could go away with to ensure the probability that at least one friend liked hip-hop music is at least 0.99? 2 marks

Question 4 (15 marks)

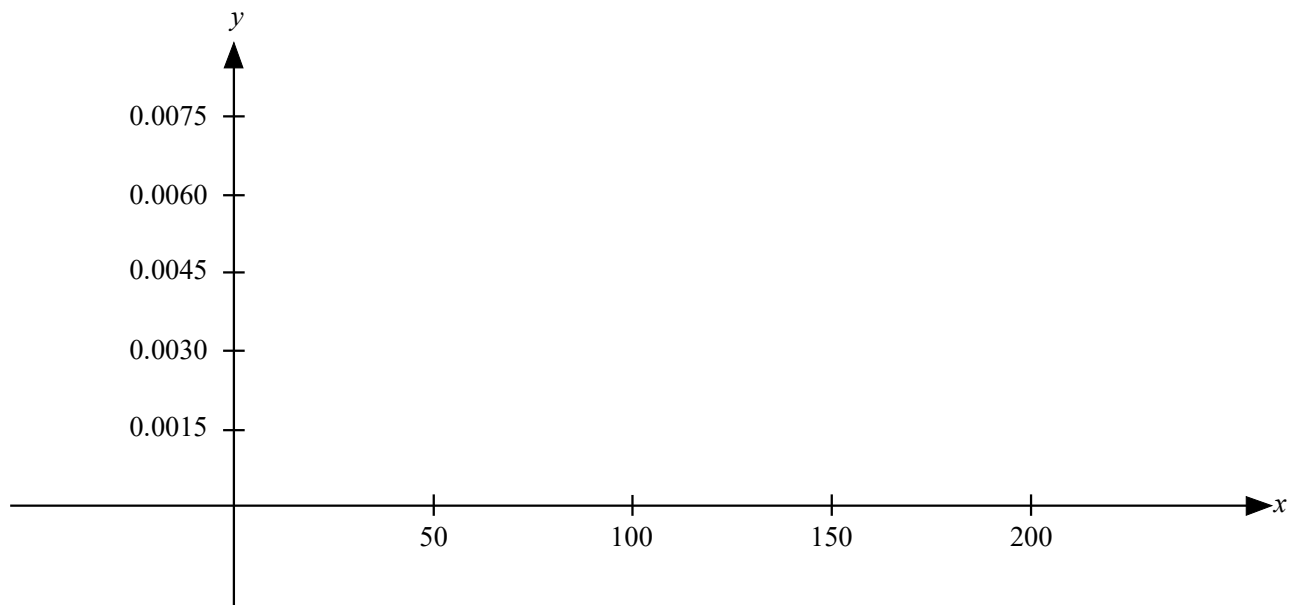
Some Scotch boys are choosing to stay in Victoria for their holiday. The distance from Melbourne, in kilometres, that a randomly selected boy who stays in Victoria will travel on his holiday is given by the continuous random variable X .

$$\text{If } f(x) = \begin{cases} k & 0 \leq x < 50 \\ \frac{70000 + 600x - 3x^2}{15000000} & 50 \leq x \leq 200 \\ 0 & \text{elsewhere} \end{cases}$$

- a.** Find the value of k such that $f(x)$ is the probability density function of random variable X . 2 marks

- b.** Sketch a graph of $y = f(x)$, showing all intercepts, turning points and endpoints on the axes below.

3 marks



- e. Find the median distance travelled by a boy who travels within Victoria. Express your answer in km correct to three decimal places.

2 marks

- f. Find the probability that a randomly chosen Scotch boy who travels within Victoria for his holiday travels at least 150 km from Melbourne, given that he travels at least the median distance from Melbourne.

2 marks

Question 5 (12 marks)

One group of boys plan to compete in the Portsea Swim Classic over summer, so will use the time after exams for some intense ocean swimming training.

The boys aim to swim together in the ocean most mornings. If the weather is fine, the probability they will all swim together is $\frac{2}{3}$. If the weather is not fine, the probability they will all swim together is $\frac{1}{4}$. Assume that weather on one morning is independent of weather on any other morning.

- a.** If the weather is fine one morning, but not fine the next morning, find the probability that the boys will swim together on at least one of those two mornings.

2 marks

- b.** In the month of November, the probability of fine weather in the morning is $\frac{7}{8}$. Find the probability that on a particular morning in November when the boys swim together, the weather will be fine.

3 marks

Question 6 (7 marks)

Another group of seven boys are planning to fly to Noosa to compete in the Hinterland to Surf Run. Online results from the previous five years of the event show that the times across all participants are normally distributed with a mean of 45 minutes and a standard deviation of 7 minutes.

- a.** Find the probability that a runner chosen at random from all participants will record a time between 35 minutes and 50 minutes. Give your answer correct to four decimal places. 2 marks

A group of 10 runners are selected at random from all participants in the Hinterland to Surf Run.

- b.** Find the probability that at least eight of these runners will record a time between 35 and 50 minutes. Give your answer correct to four decimal places. 2 marks

All seven boys will be competing in the Open Male category, where results are also normally distributed. In this category, 18% of runners record a time faster than 35 minutes, and 6% of runners record a time slower than 50 minutes.

- c.** Calculate the mean and standard deviation of the times in the Open Male category. Give times in minutes correct to four decimal places. 3 marks
