

Scotch College

	Scotch Student ID #					
	0	0	0	0		
gits	1	1	1	1		
dig	2	2	2	2		
ant	3	3	3	3		
ev	4	4	4	4		
Circle the relevant digits	5	5	5	5		
	6	6	6	6		
	7	7	7	7		
	8	8	8	8		
	9	9	9	9		

Teacher's Name

MATHEMATICAL METHODS

Unit 3-SAC 1c – Application Task: Test

WORKED

Thursday 2nd June 2022

SouTIONS

Reading Time	none
Writing Time	45 minutes

Task Sections	Marks	Your Marks
Extended Response Questions	30	
Total Marks	30	

n	~ ~	1		-	4:	on
	6-4	8	54	B 28		4311

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature:			

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

- Calculators are allowed
- Notes and/or references are not allowed.

At the end of the task

• Ensure you cease writing upon request.

Electronic Devices

Students are <u>not</u> allowed to have a mobile phone, smart watch and/or any other unauthorised electronic device in the SAC, unless it is TURNED OFF and is placed on the front teacher desk.

Question 1 (3 marks)

Consider the function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 4 \times e^{(x-2)} + 1$.

a. Find $f^{-1}(x)$.

1 mark

Let y = f(a). Suap x and y for inverse. $x = 4e^{y-2} + 1$ $\frac{x-1}{4} = e^{y-2}$

 $y = 2 + \log_2\left(\frac{\pi}{4}\right)$

:. $f^{-1}(x) = 2 + \log_{e}(\frac{x-1}{4})$

b. State the domain and range for f^{-1} .

2 marks

dom $f^{-1} = ran f = (1, \infty)$

ran f-1 = dom f = 1R

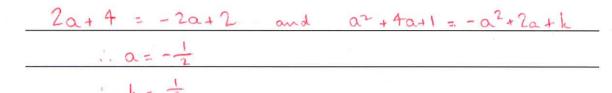
Question 2 (4 marks)

$$f(x) = \begin{cases} x^2 + 4x + 1, & x \le a \\ -x^2 + 2x + k, & x > a \end{cases}$$

The function f is continuous and joins smoothly at x = a.

a. Find the values of a and k.

3 marks



b. Find the co-ordinates of the point at which the joining occurs.

1 mark

$$S(-\frac{1}{2}) = -\frac{3}{4}$$

So $\left(-\frac{1}{2}, -\frac{3}{4}\right)$

Question 3 (5 marks)

a. Find, in terms of a, the equation of the tangent to the graph of $y = 2x^2 - 8x + 11$ at the point where x = a.

1 mark

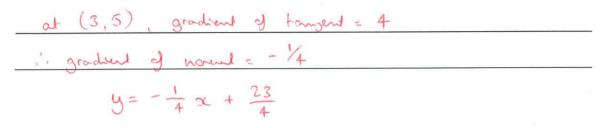
$$\frac{dy}{dx} = 4x - 8$$

$$y - (2a^2 - 8a + 11) = (4a - 8)(x - a)$$

$$y = (4a - 8)x - 2a^2 + 11$$

b. Find the equation of the normal to the graph of $y = 2x^2 - 8x + 11$ at the point (3,5).

1 mark



c. The tangent to the graph of $y = 2x^2 - 8x + 11$ at the point (b, c) is parallel to the normal to the graph of $y = 2x^2 - 8x + 11$ at the point (1,5). Find the values b and c.

3 marks

Normal ab (1,5) has gradient
$$m=\frac{1}{4}$$

is tangent at (b,c) has gradient $m=\frac{1}{4}$

$$b=\frac{33}{16}$$

$$c=\frac{385}{129}$$

Question 4 (11 marks)

The concentration (given in mL/L) of a particular medication in a person's bloodstream t minutes after it is taken is modelled by $f:[0,240] \to \mathbb{R}, f(t) = \frac{3^6 \log_{10}(t+1)}{(t+1)e^3}$.

a. Calculate the concentration of medicine in the person's blood stream 1 minute after the medicine is taken, giving your answer in mL/L correct to two decimal places.

1 mark

The medicine is effective whilst the concentration in the bloodstream is greater than 1mL/L. The concentration of medicine in the blood stream is considered toxic when greater than 15mL/L.

b. Find the maximum concentration of medicine in the bloodstream in mL/L, and the time at which this occurs in minutes. Give both answers correct to one decimal place.

2 marks

Max =
$$5.8 \text{ mL/L}$$

which occur at $b = 1.7 \text{ mins}$.

c. For how long is the medicine effective? Give your answer in minutes correct to two decimal places.

2 marks

$$\xi(t) = 1$$

$$t = 0.0702... \text{ and } t = 65.0524...$$

In an attempt to increase the effective period of the medicine to be greater than 3 hours, the	
chemicals in the medicine are changed such that the concentration in the bloodstream t minutes	
after the medicine is taken is now modelled by $g:[0,240] \to \mathbb{R}, g(t) = \frac{4^6 \log_{10}(t+1)}{(t+1)e^4}$.	
d. i. Does the model predict that the concentration of medicine in the blood stream will	
become toxic? Justify your answer.	1 mark
Maximm value ≈ 12 ml/L, so of does	
not become texte.	
ii. Does the model predict an effective period of more than 3 hours? Justify your answer. $g(t) = 1$	2 marks
L- 0.0321 0- L- 165.089	
t=0.0321 or $t=165.689t=0.0321$ or $t=165.689$	ion Shows.
It is found that by adjusting the amount of Chemical $K(k)$ in the medicine, the concentration in	
the bloodstream t minutes after the medicine is taken can be modelled by	
$h:[0,240] \to \mathbb{R}, h(t) = \frac{k^6 \log_{10}(t+1)}{(t+1)e^k}, \text{ where } k \in [2,10].$	
e. For what value of k does the concentration of the medicine in the bloodstream reach its	
maximum value?	2 marks
L'(t) =0	
t= e-1	
solve de (h(e-1))=0	
$k = 6$ as $k \in [2, 10]$	
f. For what values of k does the concentration of the medicine become toxic? Give your	

For what values of k does the concentration of the medicine become toxic? Give your answer correct to two decimal places.

1 mark

ke (4.55, 7.72)

Note: (,) as concentration must be greater than 15

Question 5 (7 marks)

a. The graphs of the equations $y_1 = 2e^{x-3} - 3$ and $y_2 = e^{2(x-3)} - 2$ intersect at a single point and have a common tangent at that point. Find the equation of the common tangent at that point. 3 marks

$$\frac{dy_1}{dx} = \frac{dy_2}{dx}$$

at
$$\alpha = 3$$
, $y_1 = -1$ and $m = 2$

$$y - (-1) = 2(x-3)$$

- **b.** Consider the graphs of the equations $y_1 = ae^{(x+1)} k$ and $y_2 = e^{a(x+1)} p$ where $k, p \in \mathbb{R}$ and $a \in \mathbb{R} \setminus \{0,1\}$. Given that the graphs intersect at a single point and have a common tangent at that point,
 - i. find p in terms of a and k so that a common tangent exists

2 marks

$$\frac{dy_1}{dx} = \frac{dy_2}{dx}$$

$$i \cdot \alpha = -1$$

at
$$\alpha = -1$$
, $y_1 = \alpha - k$ and $y_2 = 1 - p$

ii. find the equation of the common tangent in terms of a and k

1 mark

at
$$x=-1$$
, $y_1=a-k$ and $m=a$

$$y - (a - k) = a(x - -1)$$

$$y = a\alpha + 2a - k$$

equatio	equation $y = 3x + 4$ at this point, find the values of a, k and p.				
at	$\alpha=-1$, $y=1$ and $m=3$				
4° ° ° ×	$\alpha = 3$				
	k = 2				
	p = O				

iii. given that the graphs only intersect at (-1,1) and have a common tangent with

END OF SAC 1c

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \ n \neq -1$		
$\frac{d}{dx}\Big((ax+b)^n\Big) = an\Big(ax+b\Big)^n$	$b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, n \neq -1$		
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$		
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c, \ x >$	0	
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$		
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	r)	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$		
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)}$	$= a \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$			

Probability

Pr(A) = 1 - Pr(A')		$Pr(A \cup B) = 1$	$\Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$var(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Prol	bability distribution	Mean	Variance
discrete	$\Pr(X=x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_{a}^{b} f(x) dx$	$\mu = \int_{-\infty}^{\infty} x \ f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{P} = \frac{X}{n}$). 	mean	$E(\hat{P}) = p$
standard deviation	$\operatorname{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p}-z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p}+z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$