	Scotch Student ID #			
Circle the relevant digits	0	0	0	0
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	9	9	9	9

Teacher's Name

Scotch College

MATHEMATICAL METHODS

U4-SAC 1a – Application Task: Project

Date of distribution: Monday 8th August 2022

Due date: Monday 15th August 2022

Task Sections	Marks	Your Marks
Extended Response Questions	60	
Total Marks	**60**	

Remote Declaration

I declare that any work I have submitted for this VCE assessment is wholly my own, unless properly referenced or authorised for use by my teacher. I have had no assistance from any person in my home nor have I been assisted by, or given assistance to, a boy in my class or cohort unless specifically permitted to do so by my teacher. I have not used the internet or other sources to assist me in my responses unless specifically permitted by my teacher. I acknowledge my work may be reproduced, communicated, compared and archived for the purposes of detecting plagiarism and collusion.

Signature: ______________________________________

General Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this task are not drawn to scale.

Allowed Materials

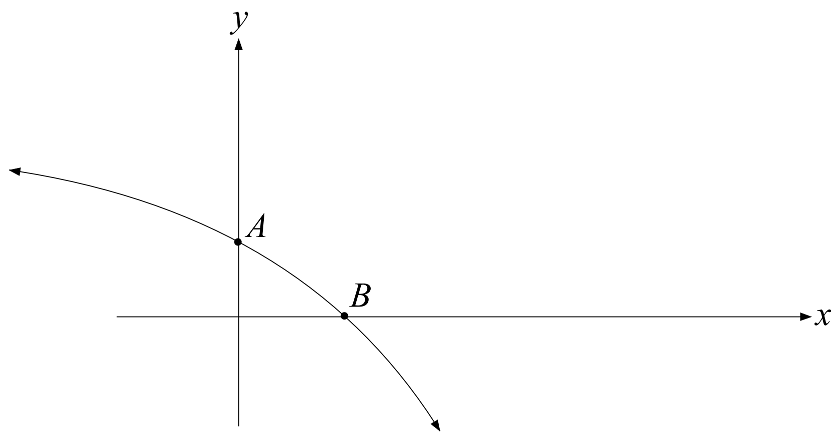
- A scientific calculator and a CAS calculator.
- Any notes or references.

At the end of the task

- Submit the task to your teacher by the due date and before the test SAC.

Question 1 (7 marks)

The function $f:\mathbb{R}\to\mathbb{R},\, f(x)=4-e^{2x}$ is shown below.



a. The function *f* crosses the *y*-axis at *A* and the *x*-axis at *B*. Find the co-ordinates of *A* and *B*. 2 marks

b. Find the area enclosed by the curve, the *x*-axis and the *y*-axis. 2 marks

c. Find the equation of the normal to the curve at *B*. 1 mark

d. Find the area of the region enclosed by the curve $y = f(x)$, the y-axis and the normal to the curve at B. 2 marks

Question 2 (6 marks)

Function g is defined such that $g(x) \geq 0$ for all x. The area of the region enclosed by the graph $y = g(x)$, the x-axis and the lines $x = 0$ and $x = 3$ is a square units.

Calculate the following, giving your answers in terms of a and the positive constant k, where appropriate.

a. $\int_3^0 g(x)\,dx$ 1 mark

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b. $\int_0^{3k} g\left(\frac{x}{k}\right)dx$ 1 mark

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c. $\int_0^3 -kg(x)\,dx$ 1 mark

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d. $\int_{3+k}^{k} g(x-k)\,dx$ 1 mark

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e. $\int_0^3 g(x)-k\,dx$ 1 mark

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f. $\int_0^{3k} g\left(\frac{x}{k}\right)+k\,dx$ 1 mark

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Question 3 (11 marks)

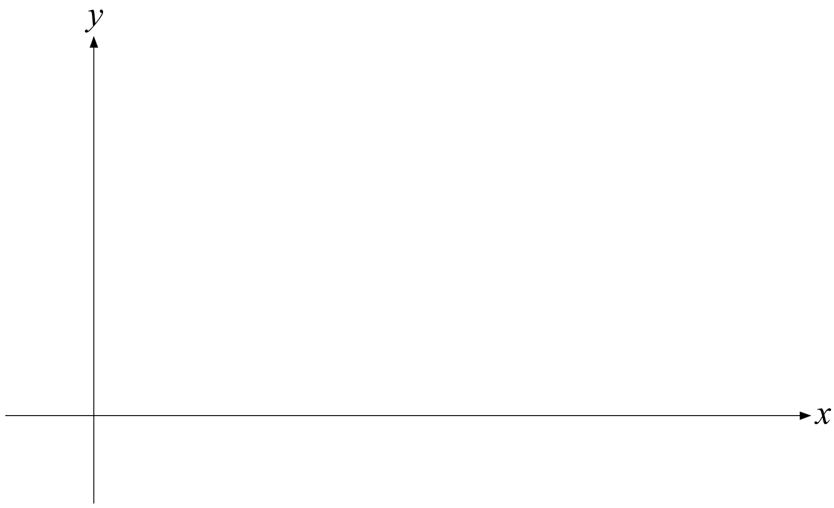
a. **i.** Function f has the rule $f(x) = a\sin(b(x+c)) + d$, where a, b, c and d are real numbers and $c \in \left[0, \frac{3\pi}{10}\right]$.

Function f has the following properties:

- Period $= \frac{2\pi}{3}$
- Range $= \left[\sqrt{3}-2, \sqrt{3}+2\right]$

Given that point $\left(\frac{\pi}{4}, 0\right)$ lies on the graph of $y = f(x)$, find the values of a, b, c and d. 3 marks

ii. Sketch the curve on the axes below, for $x \in \left[-\frac{\pi}{36}, \frac{23\pi}{36}\right]$, labelling the co-ordinates of all endpoints, intercepts and turning points. 4 marks



b. **i.** List a sequence of transformations which maps the graph of $y = \tan(x)$ to the graph of $y = \tan\left(\frac{\pi x}{2} - \frac{\pi}{3}\right)$. 2 marks

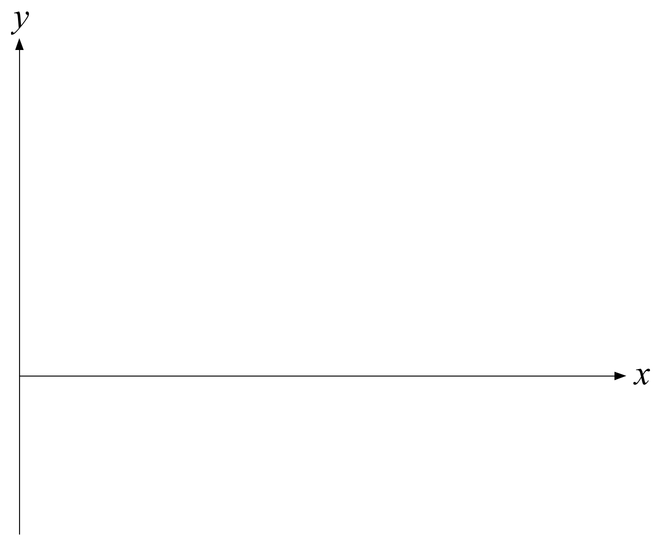
ii. The graph $y = \tan\left(\frac{\pi x}{2} - \frac{\pi}{3}\right)$ has vertical asymptotes at $x = an + b$, where $n \in \mathbb{Z}$.

Find the values of a and b. 2 marks

Question 4 (12 marks)

Function f is defined as $f:[0,4]\rightarrow \mathbb{R},\ f(x)=-2x(x-4)$.

a. Sketch the graph of $y=f(x)$ on the axes below, labelling the co-ordinates of all endpoints, intercepts and turning points. 2 marks



b. Find the area enclosed by the graph $y=f(x)$ and the x-axis. 1 mark

c. The line $y=3$ splits the region found in **part b** into two areas. Find the larger of the two areas. 3 marks

d. The normal to the curve at $x = 0.5$ splits the region found in **part b** into two areas. Find the larger of these areas. 3 marks

e. The line $y = k$ splits the region found in **part b** into two equal areas. Find the value of k. 3 marks

Question 5 (5 marks)

Function g is defined as $g:[0,4]\rightarrow\mathbb{R}, g(x)=\sqrt{x}$.

a. Find the area enclosed by the graph $y=g(x)$, the x-axis and the line $x=4$. 1 mark

b. **i.** Find the equation of the normal to $y=g(x)$ at the point $x=a$, for $a\in\left(0,\frac{7}{2}\right)$. 1 mark

ii. Find the co-ordinates of the point at which the normal to $y=g(x)$ at $x=a$ crosses the x-axis. Give your answer in terms of a. 1 mark

iii. The normal to the curve $y=g(x)$ at point $x=a$ is drawn from the point where $x=a$ to the point at which it crosses the x-axis. This divides the area enclosed by the graph $y=g(x)$, the x-axis and the line $x=4$ into two regions. Find the value of a if these two regions have equal area, giving your answer correct to two decimal places. 2 marks

Question 6 (4 marks)

a. Given that k is a constant, show that

$$\frac{d}{dx}\left(x^2\sin(2x)+2kx\cos(2x)-k\sin(2x)\right)=2x^2\cos(2x)+(2-4k)x\sin(2x).$$

1 mark

b. Using the result from **part a** with a suitable value of k, find:

$$\int_0^{\frac{\pi}{8}} x^2\cos(2x)\,dx$$

Give your answer in the form $a\sqrt{2}\left(b\pi^2+c\pi+d\right)$ where a, b, c and d are rational numbers. Show your full working.

3 marks

Question 7 (15 marks)

Function f is defined as follows:

$$f:[0,6]\to\mathbb{R},\ f(x)=4\sin\left(\frac{\pi}{3}x\right)$$

a. Write down the period and range of f. 2 marks

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b. Cubic function g has the same x-intercepts as f and passes through point $\left(\frac{3}{2},4\right)$.

Find the rule of g. 2 marks

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c. Function g has the same domain as function f. Sketch the graph of $y = g(x)$ on the axes below. Label the co-ordinates of all endpoints, intercepts and turning points. 2 marks

d. **i.** Find the co-ordinates of the points of intersection between the graphs of $y = f(x)$ and $y = g(x)$. 1 mark

ii. Hence find the total area enclosed by the graphs $y = f(x)$ and $y = g(x)$, from $x = 0$ to $x = 6$. 2 marks

e. **i.** Let $h:\left[-\frac{3}{2},\frac{9}{2}\right]\to\mathbb{R}, h(x)=4\cos\left(\frac{\pi x}{3}\right)$

State a single transformation which maps the graph of f to the graph of h. 1 mark

ii. Hence write the rule of the cubic function q with domain $[-1.5,4.5]$ with the same x-intercepts as h and the same maximum and minimum values as g. 1 mark

iii. Find the total area enclosed by the graph of $y=q(x)$ and the graph of $y=g(x)$. 2 marks

iv. Function p has the rule $p(x)=g(x+k)$, where $k\in\mathbb{R}$. Find the maximum possible area enclosed by the graphs of $y=g(x)$ and $y=p(x)$, and the value(s) of k for which this maximum area occurs. 2 marks

END OF SAC 1a

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		